Design of Power Transmission Systems

MED 411

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MED 411 The design of power transmission systems

3 Cr. hrs. = [2 Lect. + 2 Tut. + 1 Lab] Prerequisite: MED 211

Design of power Transmission Elements: Shafts, Bearings, Gears (spur, helical, bevel, and worm gears), Belts and pully, Drums and Ropes, Sprocket and chains, Brakes, Clutches, Application examples, Course Project.

مكص ٤١١ تصميم نظم نقل القدرة

التصميم التفصيلى لاجزاء نقل القدرة الميكانيكية: المحاور وكراسى المحاور، التروس العدالة والحلزونية والمائلة والدودية ، والسيور والبكرات ، واحبال الشد والجنازير ووصلات نقل الحركة الدورانية بين الاعمدة)الكبالن(والدبرياج)الكلتشات(والفرامل ، امثلة تطبيقية ، مشروع مقرر.

Laboratory:

1 Practice on simulation and animation of power transmission elements using Working Model, Adams FE and Matlab packages with practical examples.

2 Computer Oriented Manual and Automatic Transmission System (Gear Box) Design project.

References:

□ Joseph e. shigley, Charles R. Mischke, power Transmission Elements:A Mechanical Desingner,s Work book,, Mcgraw - Hill, 1990.

Assessment:

Final Exam: 50%, Quizzes: 20%, Year work: 20% ,Experimental /Oral: 10%

CHAPTER.1

Shafts, Bearings, Gears

1.1 Shafts

.-**INTODUCTIO**N

Shafts are used in all kinds of machinery and mechanical equipment to transmit power.

- Most shafts are subjected to fluctuating loads of combined bending and torsion with various degree of stress concentration.
- Beside the strengthening of the shaft, the operating speed should not be close to a critical speed.
- For more safety and smoother operating conditions shafts should be checked against deflection

2. TYPES OF SHAFTS

- (A) Transmission shafts:
- This type transmit power between the source (e.g. electric motor) and the machine absorbing power (e.g. machine tool, pump,...).

(B) Special machine shafts

- Shaft with a special construction such as the crankshafts.

From the other hand, there are circular machine elements, which remain stationary (i.e. without transmitting power) and called axles. Here, the axles is normally suffering from bending moment and normal forces without shear stress exceeded from the twisting moment.

N.B.: Spindles are shafts which transmit power but have short lengths.

3. STANDRAD SIZES OF SHAFTS

Shafts can be found as row materials in the following sizes:

- 25 mm to 60 mm with 5 mm step.
- 60 mm to 110 mm with 10 mm step.
- 110 mm to 140 mm with 15 mm step.
- 140 mm to 500 mm with 20 mm step.

The standard lengths of the row materials are: 5 m , 6 m , 7 m.

4. STRESS IN SHAFTS

According to the type of loads acting on the shafts, the shafts can rotate under the action of one of the following state of stresses:

1- Shear stress due to the transmitted power (i.e. due to the twisting torque.).

2- Normal stress due to bending moment which can be tensile or compressive. This type generated from the forces acting upon machine elements such as gears and pulley.

5. DESIGN OF SHAFTS

Design of shafts can be carried out on the basis of:

(1) Strength:

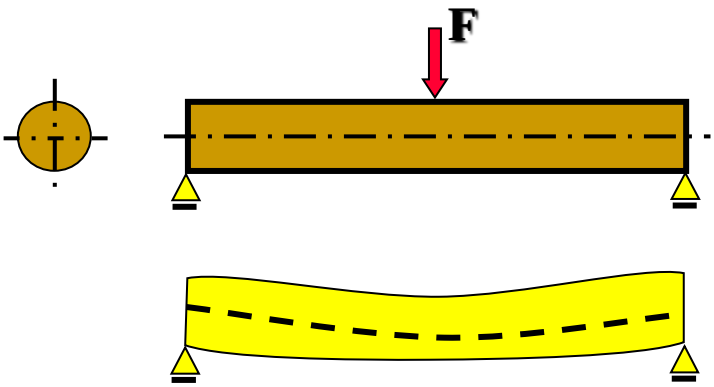
In the design proceeding according to the strength, the following conditions can be considered:

Shafts under the action of twisting moment or torque only. Shafts under the action of bending moment only or/and axial loads.

Shafts under the action of combined torsion and bending.

(2) Rigidity and Stiffness:

Here, the stability of the shafts from the point of view of deflection (in the case of bending moment) or/and buckling (in the case of compressive axial force).



Deflected Shaft

4. STRESS IN SHAFTS

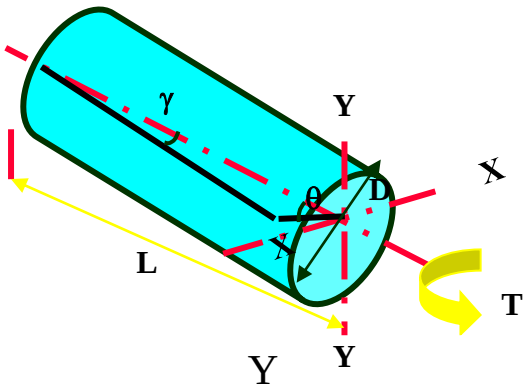
According to the type of loads acting on the shafts, the shafts can rotate under the action of one of the following state of stresses:

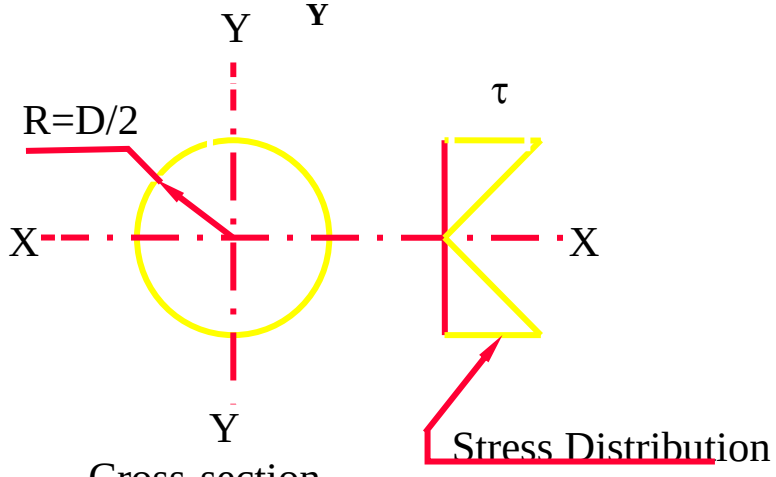
1- Shear stress due to the transmitted power (i.e. due to the twisting torque.). 2- Normal stress due to bending moment which can be tensile or compressive. This type generated from the forces acting upon machine elements such as gears and pulley.

5.1 DESIFN OF SHAFTS UNDER TWISTING MOMENT

For circular shafts under the action of twisting moment or torque, the general torsion equation can be formulated as follows:

$$\frac{T}{I_p} = \frac{\tau}{r} = \frac{\Theta G}{L}$$





$$\frac{T}{I_p} = \frac{\tau}{r} = \frac{\Theta G}{L} \qquad \text{.....(1)}$$

where:

T :The twisting moment or torque acting on the shaft in N.mm

Ip : The polar moment of inertia of the shaft in mm .

τ : The shear stress generated in the shaft in N/mm.

r : The radius of the shaft (also the position at which the shear stress generated) in mm.

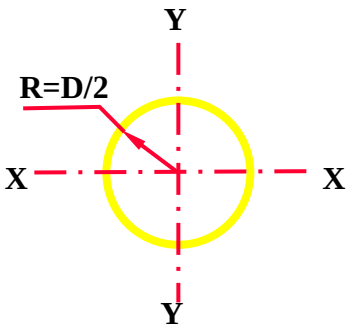
i.e. $r = d / 2$

θ : The angle of rotation of the section at which the shear stress is generated with respect to the stationary (fixed) section in radian.

CIRULAR (ROUND) SOLID SHAFTS

The polar moment of inertia can be expressed as follows:

$$I_p = \frac{\pi d^4}{32} \qquad \text{.....(2)}$$



Cross-section

$$\frac{32\,T}{\pi d^4} = \frac{\tau}{d/2} \qquad \text{...(3)}$$

From general torque equatic

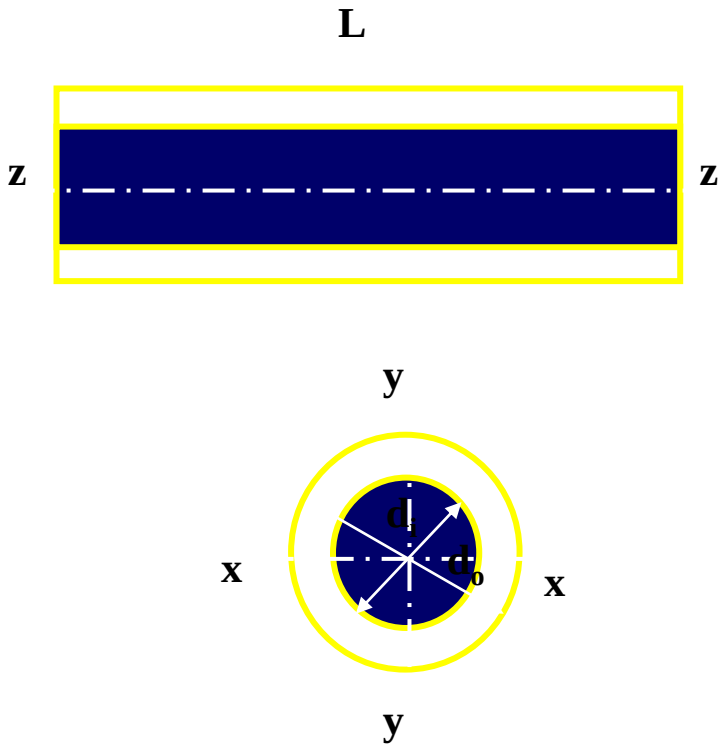
i.e.,: $$T = \frac{\pi \tau d^3}{16}$$

or : $$d = \sqrt[3]{\frac{16T}{\pi\tau}} \qquad \text{.....(4)}$$

From equation 4, the shaft diameter (d) can be determined for solid shafts subjected to a twisting moment (T).

CIRULAR (ROUND) HOLLOW SHAFTS

For hollow shafts the polar moment of inertia "Ip" can be determined from the following equation:



$$I_p = \frac{\pi}{32}\left(d_0^4 - d_i^4\right) \qquad \text{.....(5)}$$

where,

d_0 : Outer diameter of the shaft.

d_i : Inner diameter of the shaft.

Assuming that:

$$k = \frac{d_i}{d_0}$$

Then, by substituting in equation "1", the twisting moment can be expressed as follows:

$$T = \frac{\pi\tau}{16 d_0}\left(d_0^4 - d_i^4\right)$$

$$T = \frac{\pi\tau}{16 d_0} d_0^4 \left(1 - k^4\right)$$

or,

$$do = \sqrt[3]{\frac{16T}{\pi\tau(1-k^4)}} \qquad \text{……..(6)}$$

The size of a hollow shaft (i.e., determination of d_o & d_i) subjected to twisting moment can be determined from the last equation after assuming the diameters ratio (k).

<u>It must be noticed that:</u>

1. Hollow shafts are usually used in marine work (high transmitted power).
2. Hollow shafts are more stable against deflection than solid shafts.

The twisting moment (T) can be determined from the relation which connect the power with both of the twisting moment and the angular velocity as follows:

$$P = T \cdot \omega \qquad (7)$$

i.e., $$P = \frac{2\pi NT}{4500} \Rightarrow T = \frac{4500\,P}{2\pi N}$$

where;

T : Twisting moment in kp.m

N : Speed in r.p.m.

P :Transmitting power in Horse power (hp)

N.B. : 1 kW = 1.34 hp

EXAMPLE:

Shaft transmits:

- A power of 5 kW

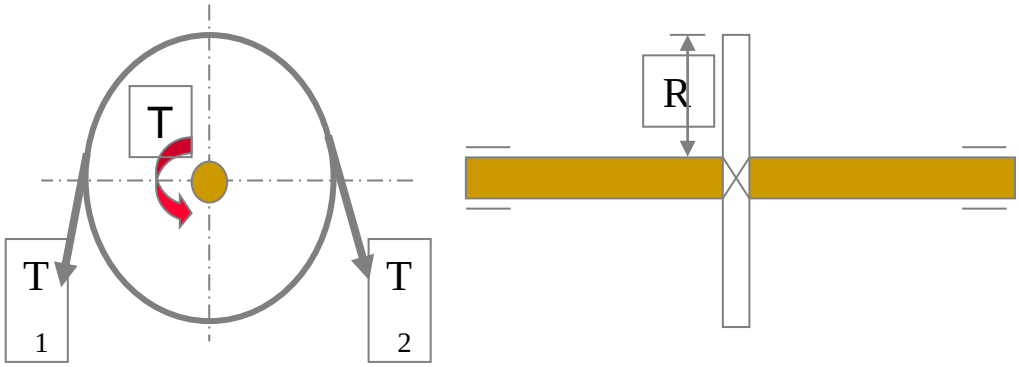
- At 500 rpm

P = 5 kw = 5 x 1.34 = 6.7 HP

$$T = \frac{4500\,P}{2\pi N}$$

$$T = \frac{4500 \times 6.7}{2\pi \times 500} = 9.6 \ \text{kp.m} = 96 \ \text{Nm}$$

Also, in the case of using belt drive unites, the transmitting torque can be computed from the following relation:



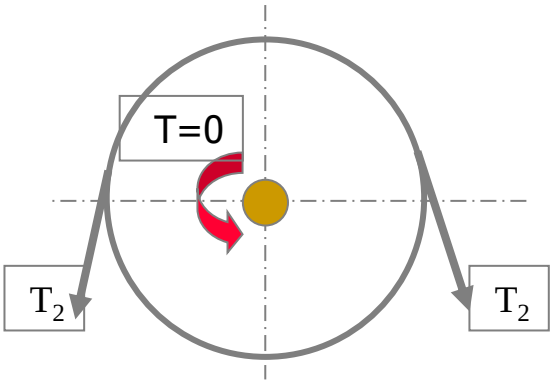
$$T = (T_1 - T_2)\,R \qquad (8)$$

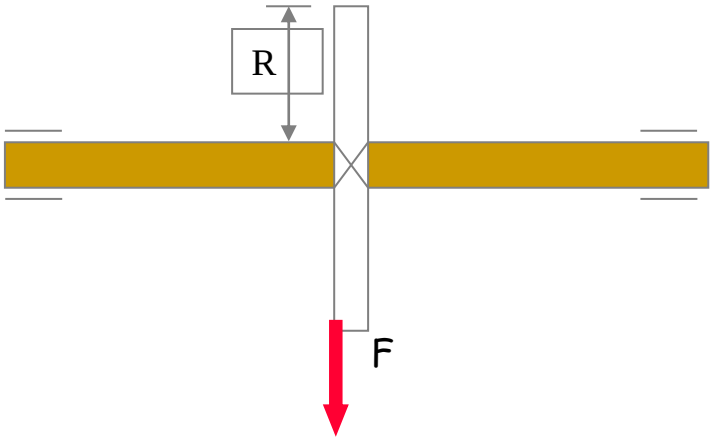
where;

T1 and T2 are the tensions on the tight side and slack side of the belt respectively and (R) is the radius of the pulley.

IMPORTANT NOTCE:

For stationary pulley condition i.e. T = 0





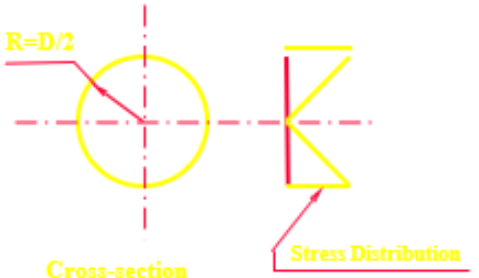
$F = 2\ T_2$ Stationary condition

$F = T_1 + T_2$ Running condition

SUMMARY (SHAFT UNDER SHEAR STRESS)

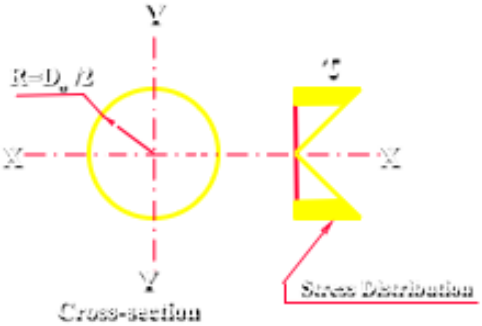
Solid shaft

$$d = \sqrt[3]{\frac{16T}{\pi\tau}}$$



Hollow shaft

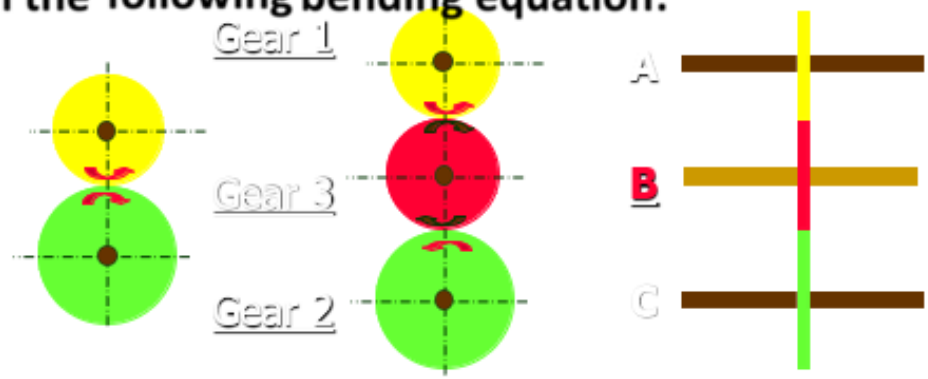
$$do = \sqrt[3]{\frac{16T}{\pi\tau(1-k^4)}}$$

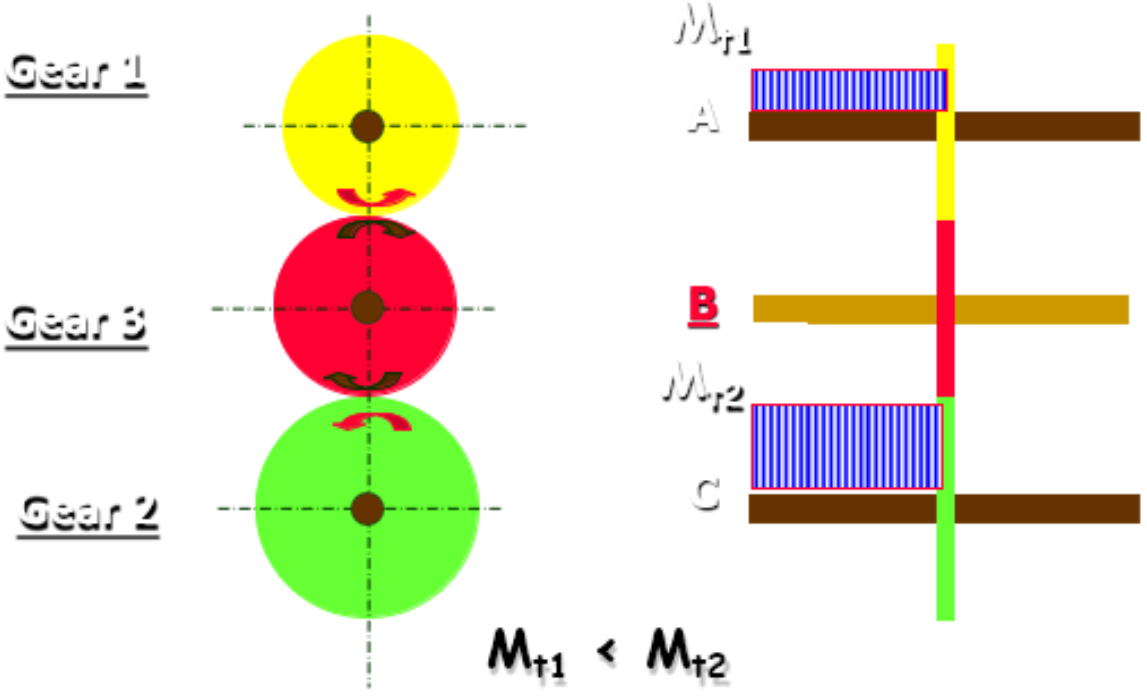


$$P = \frac{2\pi NT}{4500} \qquad T = \frac{4500\ P}{2\pi N}$$

The shafts; which rotate under the action of pure bending moment; are subjected to a uni-axial state of tensile and compressive stress.

This also the case of the axles, which transmit bending moment only The design of both of them should be based on the following bending equation:

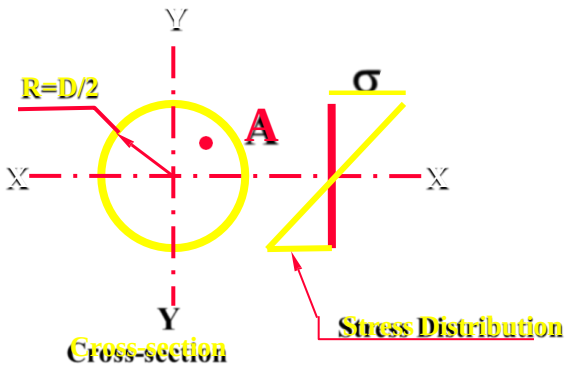




$M_{t1} < M_{t2}$

$N_1 > N_2$

$$\sigma_b = \frac{My}{I}$$



where;

M :Maximum bending moment acting on the critical section in N.mm

I : Second moment of inertia of the cross section about the axis of rotation in mm^4 .

σ_b : Normal stress due to bending moment in kp/cm

y : The distance from the neutral axis to the outer most fiber on which the stress is maximal in mm.

For solid circular shaft:

$$I = \frac{\pi}{64} d^4 \qquad y = \frac{d}{2}$$

$$\therefore \sigma_b = \frac{M_b (d/2)}{\frac{\pi d^4}{64}}$$

Therefore, the diameter of solid shafts can be determined from the following equation:

$$d = \sqrt[3]{\frac{32 M_b}{\pi \sigma_b}}$$

For hollow circular shaft:

$$I = \frac{\pi}{64}(d_o^4 - d_i^4) \qquad \& \qquad y = \frac{d_o}{2}$$

$$\therefore \sigma_b = \frac{M_b(d/2)}{\frac{\pi}{64}(d_o^4 - d_i^4)} \quad \text{or,} \quad \sigma_b = \frac{M_b(d_o/2)}{\frac{\pi d_o^4}{64}(1 - k^4)}$$

Therefore, the diameter of solid shafts can be determined from the following equation:

$$d_o = \sqrt[3]{\frac{32M_b}{\pi\sigma_b(1-k^4)}}$$

$$k = \frac{d_i}{d_o} \text{ i.e., } k < 1$$

5.3 SHAFTS SUBJECTED TO COMBINED STATE OF STRESS

- In most practical cases, the shafts are subjected in general into a combination of bending and twisting moments as well as normal force.
- The design of these shafts should be based on the simultaneously effect of all the loading elements.
- The theories of elastic failure must be carefully utilized. According to practical experience, the following two theories are important in the design of these shafts:
  1. Maximum shear stress theory of elastic failure (Guest`s theory)
  2. Maximum normal stress theory (Rankine`s theory).

Maximum Shear Theory

According to the maximum shear stress theory of elastic failure, the maximum shear stress in the shaft can be expressed as follows:

$$\tau_{s\,(max)} = \frac{1}{2}\sqrt{\sigma_b^2 + 4\tau_s^2}$$

Where,

τ_s : Shear stress due to twisting moment.

σ_b : Normal stress due to bending moment.

Replacing the normal and shear stresses with their equations, which are functions of the shaft diameter and the loads (i.e. bending and twisting moments), the following relation can be obtained:

$$\tau_{(max)} = \frac{1}{2}\sqrt{\left[\frac{32M}{\pi d^3}\right]^2 + 4\left[\frac{16T}{\pi d^3}\right]^2}$$

$$= \frac{16}{\pi d^3}\sqrt{(M^2) + (T^2)}$$

$$T_e = \sqrt{M^2 + T^2}$$

T_e is known as the equivalent twisting moment.

The following table summarize the equations used in design of shaft using the allowable stress as material property.

Method	Solid Shaft	Hollow Shaft
Pure Shear Stress (M_t)	$d = \sqrt[3]{\frac{16T}{\pi \tau_{max}}}$	$do = \sqrt[3]{\frac{16T}{\pi d(1-k^4)}}$
Pure Normal Stress (M_b)	$d = \sqrt[3]{\frac{32M_b}{\pi \sigma_b}}$	$d = \sqrt[3]{\frac{32M_b}{\pi \sigma_b (1-k^4)}}$
Max. Shear Theory (M_b &M_t)	$d = \sqrt[3]{\frac{\sqrt{M^2+T^2}16}{\pi \tau_{max}}}$	$d = \sqrt[3]{\frac{\sqrt{M^2+T^2}16}{\pi \tau_{max}(1-k^4)}}$
Principle Normal Stress Theory (M_b &M_t)	$d = \sqrt[3]{\frac{32M_e}{\pi \sigma_{yt}}}$	$d_o = \sqrt[3]{\frac{32M_e}{\pi \sigma_{yt}(1-k^4)}}$

A line shaft rotates at 200 r.p.m. and transmits 25 HP. The shaft made of mild steel with an allowable shear stress of 420 kp/cm². Determine the diameter of the shaft neglecting the bending moment on it.

SOLUTION

Given

N = 200 r.p.m.

P = 25 h.p.

$(\tau_{all}) = 420$ kp/cm²

1. Determination of thetransmitted torque:

Let T = Torque transmitted by the shaft

$$T = \frac{4500P}{2\pi N}$$

$$T = \frac{25x4500}{2\pi x200} = 89.5 \text{ kp.m}$$

$$T = 8950 \text{ kp.cm}$$

T=8950 kp.cm

I. II. Diameter of the shaft

Let d = shaft diameter

$$d = \sqrt[3]{\frac{16T}{\pi\tau_{all}}}$$

$$d = \sqrt[3]{\frac{16x8950}{\pi x420}} = 4.77\text{cm}$$

i.e. d =47.7 mm

Therefore, d = 50 mm

Find the diameter of a solid steel shaft transmits 25 HP at 200 r.p.m. The ultimate shear stress is 3600 kp/cm^2 and a factor of safety as 8 .

If a hollow shaft is assumed to replace it, find the inside and outside diameters when a ratio of inside diameter to outside diameter of 0.5 is assumed.

SOLUTION

Given

Horse power transmitted,

Speed of shaft, Ultimate shear stress,

Factor of safety

=25 h.p.P

=200 r.p.m.N

=3600 kp/cm2τ_u

= 8n

Allowable shear stress, τ_{all} = 3600/8 =450 kp/cm2

I. **Determination of thetransmitted torque:**

Let T = Torque transmitted by the shaft

$$T = \frac{4500P}{2\pi N}$$

$$T = \frac{25x4500}{2\pi x200} = 89.5$$

$$T - 8950 kp.cm$$

T=8950 kp.cm

II. Diameter of the shaft

Let d = shaft diameter

$$d = \sqrt[3]{\frac{16T}{\pi \tau_{all}}}$$

$$d = \sqrt[3]{\frac{16x8950}{\pi x420}} = 4.77cm$$

i.e. **d =47.7 mm**

Therefore, **d = 50 mm**

III. Diameters of hollow shaft:

Let d_i = Inside diameter

d_o = Outside diameter

$$T_e = \frac{\pi}{16}\tau d_o^3(1-k^4)$$

$$d_o = \sqrt[3]{\frac{16T}{\pi\tau(1-k^4)}}$$

$$d_0 = \sqrt{\frac{16x8950}{\pi x450\left[1-(0.5)^4\right]}}$$

$$d_0 = 4.76 = 5cm$$

but, k = 0.5

Then,

$$d_i = 0.5 \times 50 = 25 \text{ mm}$$

Therefore,

d_o = 50 mm

d_i = 25 mm

6. SHAFTS SUBJECTED TO FLUCTUATING LOADS

In practices, shafts are subjected to fluctuating torque and bending moment. In this case, the design should be based on the consideration of the combined shock and fatigue factors.

Thus for a shaft Subjected to combined bending and torsion, the equivalent twisting moment can be expressed as follows:

$$T_e = \sqrt{(K_m M)^2 + (K_t T)^2}$$

Also, the equivalent bending moment is as follows:

$$M_e = \frac{1}{2}[K_m M + \sqrt{(K_m M)^2 + (K_t T)^2}$$

where;

K_m = Combined shock and fatigue factor for bending moment.

K_t = Combined shock and fatigue factor for twisting moment.

The recommended values for these factors are given in the following table:

Nature of Load	K_m	K_t
1. Stationary shafts:	1.0	1.0
(a)Gradually applied load	1.5 : 2.0	1.5 : 2.0
(b)Suddenly applied load	1.5	1.5
	1.5 : 2.0	1.5 : 2.0
	2.0 : 3.0	2.0 : 3.0
2. Rotating shafts:		
(a) Gradually applied load		
(b) Suddenly applied load with minor shock. (c)Suddenly applied load with major shock.		

1.2.Bearings

INTRODUCTION :

In rolling contact bearings, the contact between the bearing surfaces is rolling instead of sliding as in sliding contact bearings. We have already discussed that the ordinary sliding bearing starts from rest with practically metal-to-metal contact and has a high coefficient of friction. It is an outstanding advantage of a rolling contact bearing over a sliding bearing that it has a low starting friction.

TYPES OF ROLLING CONTACT BEARINGS

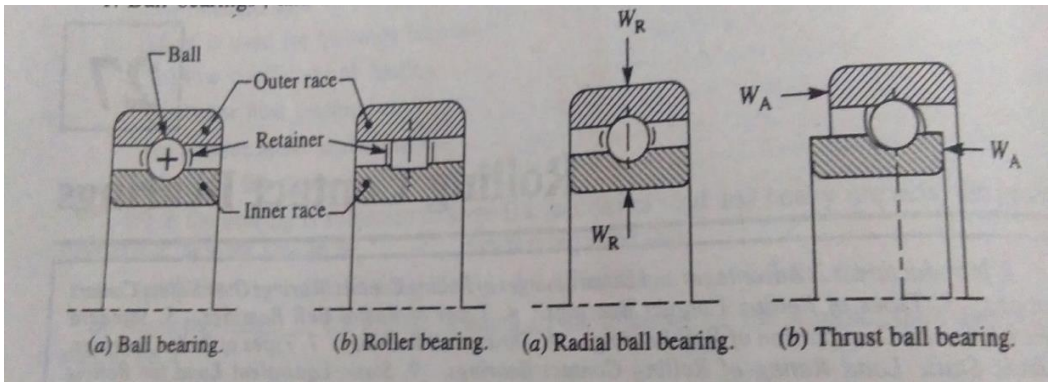
Following are the two types of rolling contact bearings:

1. Ball bearings:
2. Roller bearings.

The ball and roller bearings consist of an inner race which is mounted on the shaft or journal and an outer race which is carried by the housing or casing. In between the inner and outer race, number of balls or rollers are used and these are held at proper distances by retainers so that they do not touch each other. The retainers are thin strips and used for the balls have been properly spaced.

The ball bearings are used for light loads and the roller bearings are used for heavier loads. The rolling contact bearings, depending upon the load to be carried, are classified as :

(a) Radial bearings, and (b) Thrust bearings.



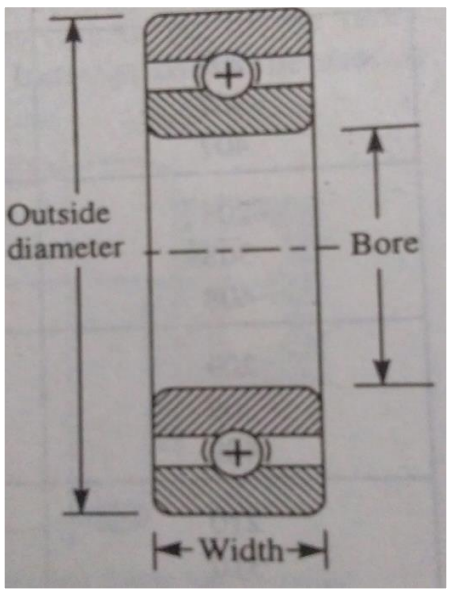
SLIDING CONTACT BEARINGS: ADVANTAGES

1. **Low starting and running friction except at very high speeds.**
2. **Ability to withstand momentary shock loads.**
3. **Accuracy of shaft alignment.**
4. **Low cost of maintenance, as no lubrication is required while in service.**
5. **Small overall dimensions.**
6. **Reliability of service.**
7. **Easy to mount and erect.**
8. **Cleanliness.**

DISADVANTAGES

1. **More noisy at very high speeds.**
2. **Low resistance to shock loading.**
3. **More initial cost.**
4. **Design of bearing housing complicated.**

STANDARD DIMENSIONS AND DESIGNATIONS OF BALL BEARINGS:



The dimensions that have been standardised on an international basis are shown in Fig. These dimensions are a function of the bearing bore and the series of bearing.

The standard dimensions are given in millimetres. There is o standard for the size and number of steel balls.

The bearings are designated by a number. In general, the number consists of atleast three digits. Additional digits or letters are used to indicate special features e.g. deep groove, filling notch etc.

The last three digits give the series and the bore of the bearing. The last two digits from 04 onwards, when multiplied by 5, give the bore diameter in millimetres. The third from the last digit designates the series of the bearing.

The most common ball bearings are available in four series as follows :

1. Extra light (100), 2. Light (200), 3. Medium (300), 4. Heavy (400)

BASIC STATIC LOAD RATING OF ROLLING CONTACT BEARINGS

The load carried by a non-rotating bearing is called a static load. The basic static load rating is defined as the static radial load (in case of radial ball or roller bearings) or axial load (in case of thrust ball or roller bearings) which corresponds to a total permanent deformation of the ball (or roller) and race, at the most heavily stressed contact, equal to 0.0001 times the ball (or roller) diameter. In single row angular contact ball bearings, the basic static load relates to the radial component of the load, which causes a purely radial displacement of the bearing rings in relation to each other.According to IS : 3823–1984, the basic static load rating (C0) in newtons for ball and roller bearings may be obtained as discussed below :

1. **For radial ball bearings, the basic static radial load rating (C0) is given by** $C_0 = f_0 . i . Z . D^2 \cos \alpha$

Where,

i = Number of rows of balls in any one bearing, Z = Number of ball per row,

D = Diameter of balls, in mm,

α = Nominal angle of contact i.e. the nominal angle between the line of action of the ball load and a plane perpendicular to the axis of bearing, and f_0 **= A factor depending upon the type of bearing.**

The value of factor (f 0) for bearings made of hardened steel are taken as follows :
f 0 = 3.33, for self-aligning ball bearings = 12.3, for radial contact and angular contact groove ball bearings.

For radial roller bearings, the basic static radial load rating is given by $C_0 = f_0.i.Z.l_e.D\cos\alpha$
Where,
i = Number of rows of rollers in the bearing, Z = Number of rollers per row,

l_e = Effective length of contact between one roller and that ring

D = Diameter of roller in mm. It is the mean diameter in case of tapered rollers,

α = Nominal angle of contact. It is the angle between the line of action of the roller resultant load and a plane perpendicular to the axis of the bearing, and

f_0 = 21.6, for bearings made of hardened steel.

3For thrust ball bearings,

The basic static axial load rating is given by

$$C_0 = f_0.Z.D^2 \sin\alpha$$

where

Z = Number of balls carrying thrust in one direction, and f_0 = 49, for bearings made of hardened steel.

4 For thrust roller bearings,

The basic static axial load rating is given by

where

$C_0 = f_0 . Z . l_e . D . \sin \alpha$

Z = Number of rollers carrying thrust in one direction, and

f 0 = 98.1, for bearings made of hardened steel.

STATIC EQUIVALENT LOAD FOR ROLLING CONTACT BEARINGS

The static equivalent load may be defined as the static radial load (in case of radial ball or roller bearings) or axial load (in case of thrust ball or roller bearings) which, if applied, would cause the same total permanent deformation at the most heavily stressed ball (or roller) and race contact as that

which occurs under the actual conditions of loading. The static equivalent radial load (W0R) for radial or roller bearings under combined radial and axial or thrust loads is given by the greater magnitude of those obtained by the following two equations, i.e.

$W_{0R} = X_0 . W_R + Y_0 . W_A$;

X0 = Radial load factor, and

Y0 = Axial or thrust load factor.

According to IS : 3824 – 1984, the values of X0 and Y0 for different bearings are given in the **databook table**

S. No.	Type of bearing	Single row bearing		Double row bearing	
		X_0	Y_0	X_0	Y_0
1.	Radial contact groove ball bearings.	0.60	0.50	0.60	0.50
2.	Self aligning ball or roller bearings and tapered roller bearing.	0.50	0.22 cot θ	1	0.44 cot θ
3.	Angular contact groove bearings :				
	α = 15°	0.50	0.46	1	0.92
	α = 20°	0.50	0.42	1	0.84
	α = 25°	0.50	0.38	1	0.76
	α = 30°	0.50	0.33	1	0.66
	α = 35°	0.50	0.29	1	0.58
	α = 40°	0.50	0.26	1	0.52
	α = 45°	0.50	0.22	1	0.44

LIFE OF A BEARING

The life of an individual ball (or roller) bearing may be defined as the number of revolutions (or hours at some given constant speed) which the bearing runs before the first evidence of fatigue develops in the material of one of the rings or any of the rolling elements. The rating life of a group of apparently identical ball or roller bearings is defined as the number of revolutions (or hours at some given constant speed) that 90 per cent of a group of bearins will complete or exceed before the first evidence of fatigue develops.

The term minimum life is also used to denote the rating life. It has been found that the life which 50 per cent of a group of bearings will complete or exceed is approximately 5 times the life which 90 per cent of the bearings will complete or exceed. In other words, we may say that the average life of a bearing is 5 times the rating life (or minimum life).

It may be noted that the longest life of a single bearing is seldom longer than the 4 times the average life and the maximum life of a single bearing is about 30 to 50 times the minimum life.

DYNAMIC LOAD RATING OF ROLLING CONTACT BEARINGS

The basic dynamic load rating is defined as the constant stationary radial load (in case of radial ball or roller bearings) or constant axial load (in case of thrust ball or roller bearings) which a group of apparently identical bearings with stationary outer ring can endure

for a rating life of one million revolutions with only 10 per cent failure. The basic dynamic load rating (C) in newtons for ball and roller bearings may be obtained as discussed below :

1. According to IS: 3824 (Part 1)– 1983, the basic dynamic radial load rating for radial and angular contact ball bearings, except the filling slot type, with balls not larger than 25.4 mm in diameter, is given by

$$C = f_C \, (i \cos \alpha)^{0.7} \, Z^{2/3} \, . \, D^{1.8}$$

and for balls larger than 25.4 mm in diameter,

$$C = 3.647 \, f_C \, (i \cos \alpha)^{0.7} \, Z^{2/3} \, . \, D^{1.4}$$

DYNAMIC EQUIVALENT LOAD FOR ROLLING CONTACT BEARINGS

The dynamic equivalent load may be defined as the constant stationary radial load (in case of radial ball or roller bearings) or axial load (in case of thrust ball or roller

bearings) which, if applied to a bearing with rotating inner ring and stationary outer ring, would give the same life as that which the bearing will attain under the actual conditions of load and rotation.

The dynamic equivalent radial load (W) for radial and angular contact bearings, except the filling slot types, under combined constant radial load (WR) and constant axial or thrust load (WA) is given by

$$W = X \cdot V \cdot WR + Y \cdot WA$$

where V = A rotation factor, = 1, for all types of bearings when the inner race is rotating, = 1, for self-aligning bearings when inner race is stationary,

= 1.2, for all types of bearings except self-aligning, when inner race is stationary.

The values of radial load factor (X) and axial or thrust load factor (Y) for the dynamically loaded bearings may be taken from Data Table:

DYNAMIC LOAD RATING FOR ROLLING CONTACT BEARINGS UNDER VARIABLE LOADS

The approximate rating (or service) life of ball or roller bearings is based on the fundamental equation,

$$\mathbf{L = (C/W)^k \times 10^6 \quad revolutions}$$

Or $\mathbf{C = W (L/10^6)^{1/k}}$

Where

L = Rating life, and

C = Basic dynamic load rating, W

= Equivalent dynamic load,

k = 3, for ball bearings, = 10/3, for roller bearings.

The relationship between the life in revolutions (L) and the life in working hours (LH) is given by

$$L = 60\ N \cdot LH \quad \text{revolutions}$$

MATERIALS AND MANUFACTURE OF BALL AND ROLLER BEARINGS

Since the rolling elements and the races are subjected to high local stresses of varying magnitude with each revolution of the bearing, therefore the material of the rolling element (i.e. steel) should be of high quality.

The balls are generally made of high carbon chromium steel. The material of both the balls and races are heat treated to give extra hardness and toughness. The balls are manufactured by hot forging on hammers from steel rods. They are then heattreated, ground and polished. The races are also formed by forging and then heat-treated, ground and polished.

LUBRICATION OF BALL AND ROLLER BEARINGS

The ball and roller bearings are lubricated for the following purposes :

1. To reduce friction and wear between the sliding parts of the bearing,
2. To prevent rusting or corrosion of the bearing surfaces,
3. To protect the bearing surfaces from water, dirt etc., and
4. To dissipate the heat. In general, oil or light grease is used for lubricating ball and roller bearings. Only pure mineral oil or a calcium-base grease should be used. If there is a possibility of moisture contact, then potassium or sodium-base greases may be used.

Bearing Load Life at Rated Reliability

When nominally identical groups are tested to the life-failure criterion at different loads, the data are plotted on a graph as depicted in Fig. 11–4 using a log-log transformation. To establish a single point, load F_1 and the rating life of group one $(L_{10})_1$ are the coordinates that are logarithmically transformed. The reliability associated with this point, and all other points, is 0.90. Thus we gain a glimpse of the load-life function at 0.90 reliability. Using a regression equation of the form

$$FL^{1/a} = \text{constant} \tag{11–1}$$

the result of many tests for various kinds of bearings result in

- $a = 3$ for ball bearings
- $a = 10/3$ for roller bearings (cylindrical and tapered roller)

A *catalog load rating* is defined as the radial load that causes 10 percent of a group of bearings to fail at the bearing manufacturer's rating life. We shall denote the catalog load rating as C_{10}. The catalog load rating is often referred to as a *Basic Dynamic Load Rating*, or sometimes just Basic Load Rating, if the manufacturer's rating life is 10^6 revolutions. The radial load that would be necessary to cause failure at such a low life would be unrealistically high. Consequently, the Basic Load Rating should be viewed as a reference value, and not as an actual load to be achieved by a bearing.

In selecting a bearing for a given application, it is necessary to relate the desired load and life requirements to the published catalog load rating corresponding to the catalog rating life. From Eq. (11–1) we can write

$$F_1 L_1^{1/a} = F_2 L_2^{1/a} \tag{11–2}$$

where the subscripts 1 and 2 can refer to any set of load and life conditions. Letting F_1 and L_1 correlate with the catalog load rating and rating life, and F_2 and L_2 correlate with desired load and life for the application, we can express Eq. (11–2) as

$$F_R L_R^{1/a} = F_D L_D^{1/a} \tag{a}$$

where the units of L_R and L_D are revolutions, and the subscripts R and D stand for Rated and Desired.

It is sometimes convenient to express the life in hours at a given speed. Accordingly, any life L in revolutions can be expressed as

$$L = 60\,\mathcal{L}n \tag{b}$$

where $\mathcal{L}$ is in hours, n is in rev/min, and 60 min/h is the appropriate conversion factor.

Incorporating Eq. (b) into Eq. (a),

$$F_R(\mathcal{L}_R n_R 60)^{1/a} = F_D(\mathcal{L}_D n_D 60)^{1/a} \tag{c}$$

catalog rating, lbf or kN

rating life in hours

rating speed, rev/min

desired speed, rev/min

desired life, hours

desired radial load, lbf or kN

Solving Eq. (c) for F_R, and noting that it is simply an alternate notation for the catalog load rating C_{10}, we obtain an expression for a catalog load rating as a function of the desired load, desired life, and catalog rating life.

$$C_{10} = F_R = F_D\left(\frac{L_D}{L_R}\right)^{1/a} = F_D\left(\frac{\mathcal{L}_D n_D 60}{\mathcal{L}_R n_R 60}\right)^{1/a} \tag{11–3}$$

It is sometimes convenient to define $x_D = L_D/L_R$ as a dimensionless *multiple of rating life*.

Relating Load, Life, and Reliability

This is the designer's problem. The desired load is not the manufacturer's test load or catalog entry. The desired speed is different from the vendor's test speed, and the reliability expectation is typically much higher than the 0.90 accompanying the catalog entry. Figure 11–5 shows the situation. The catalog information is plotted as point A, whose coordinates are (the logs of) C_{10} and $x_{10} = L_{10}/L_{10} = 1$, a point on the 0.90 reliability contour. The design point is at D, with the coordinates (the logs of) F_D and x_D, a point that is on the $R = R_D$ reliability contour. The designer must move from point D to point A via point B as follows. Along a constant reliability contour (BD), Eq. (11–2) applies:

$$F_B x_B^{1/a} = F_D x_D^{1/a}$$

from which

$$F_B = F_D \left(\frac{x_D}{x_B}\right)^{1/a} \qquad (a)$$

Along a constant load line (AB), Eq. (11–4) applies:

$$R_D = \exp\left[-\left(\frac{x_B - x_0}{\theta - x_0}\right)^b\right]$$

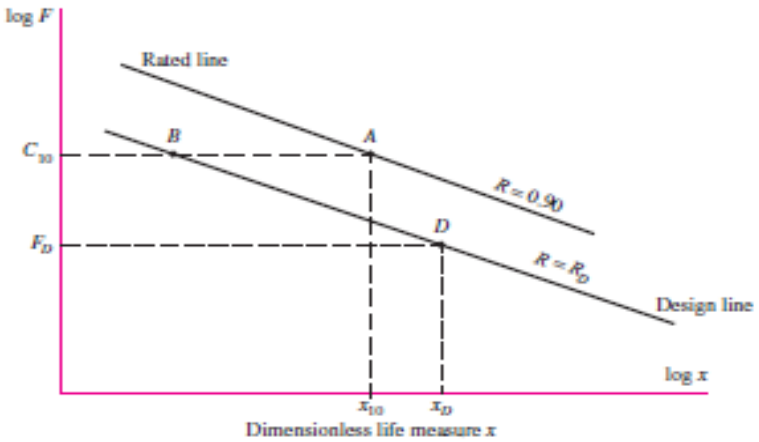


Figure 11–5

Constant reliability contours. Point A represents the catalog rating C_{10} at $x = L/L_{10} = 1$. Point B is on the target reliability design line R_D, with a load of C_{10}. Point D is a point on the desired reliability contour exhibiting the design life $x_D = L_D/L_{10}$ at the design load F_D.

$$C_{10} \doteq a_f F_D \left[\frac{x_D}{x_0 + (\theta - x_0)(1 - R_D)^{1/b}}\right]^{1/a} \qquad R \geq 0.90 \qquad (11\text{–}7)$$

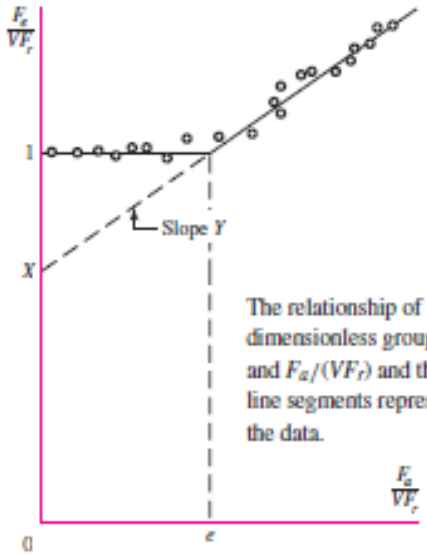
Either Eq. (11–6) or Eq. (11–7) may be used to convert from a design situation with a desired load, life, and reliability to a catalog load rating based on a rating life at 90 percent reliability. Note that when $R_D = 0.90$, the denominator is equal to one, and the equation reduces to Eq. (11–3). The Weibull parameters are usually provided in the manufacturer's catalog. Typical values are given on p. 608 at the beginning of the end-of-chapter problems.

Combined Radial and Thrust Loading

A ball bearing is capable of resisting radial loading and a thrust loading. Furthermore, these can be combined. Consider F_a and F_r to be the axial thrust and radial loads, respectively, and F_e to be the *equivalent radial load* that does the same damage as the combined radial and thrust loads together. A rotation factor V is defined such that $V = 1$ when the inner ring rotates and $V = 1.2$ when the outer ring rotates. Two dimensionless groups can now be formed: F_e/VF_r and F_a/VF_r. When these two dimensionless groups are plotted as in Fig. 11–6, the data fall in a gentle curve that is well approximated by two straight-line segments. The abscissa e is defined by the intersection of the two lines. The equations for the two lines shown in Fig. 11–6 are

$$\frac{F_e}{VF_r} = 1 \qquad \text{when } \frac{F_a}{VF_r} \le e \tag{11–8a}$$

$$\frac{F_e}{VF_r} = X + Y\frac{F_a}{VF_r} \qquad \text{when } \frac{F_a}{VF_r} > e \tag{11–8b}$$



The relationship of dimensionless group $F_e/(VF_r)$ and $F_a/(VF_r)$ and the straight-line segments representing the data.

$$F_e = X_i V F_r + Y_i F_a \tag{11–9}$$

where $i = 1$ when $F_a/VF_r \le e$ and $i = 2$ when $F_a/VF_r > e$. The X and Y factors depend upon the geometry and construction of the specific bearing. Table 11–1 lists representative values of X_1, Y_1, X_2, and Y_2 as a function of e, which in turn is a function of F_a/C_0, where C_0 is the basic static load rating. The *basic static load rating* is the load that will produce a total permanent deformation in the raceway and rolling element at any contact point of 0.0001 times the diameter of the rolling element. The basic static load rating is typically tabulated, along with the basic dynamic load rat-

(PART B)

SLIDING CONTACT BEARING

If the rubbing surfaces of a bearing are in direct contact without any rolling elements, there will be rapid wear. In order to reduce frictional resistance and wear and in some cases to carry away the heat generated, a layer of fluid (known as lubricant) may be provided.

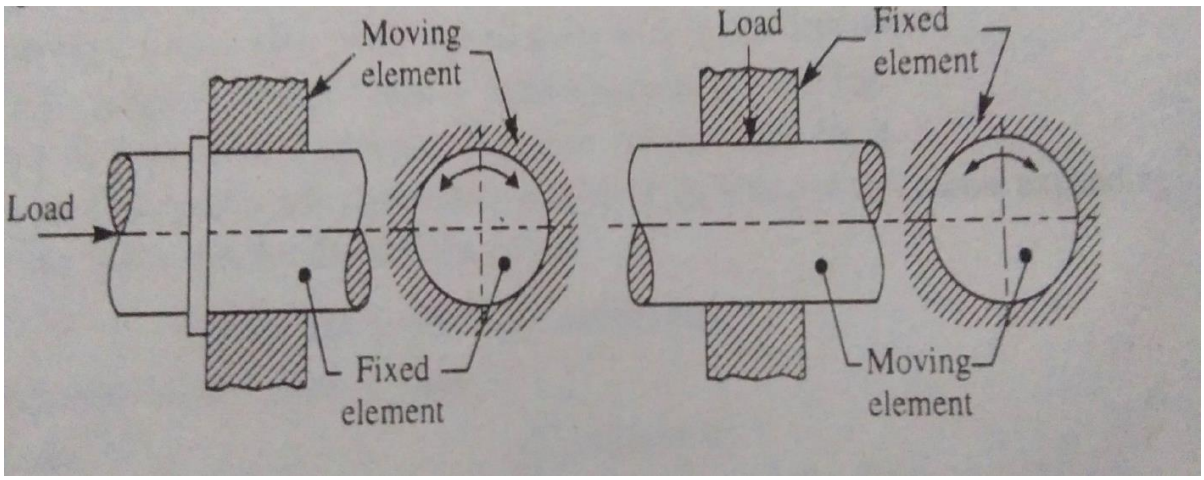
The lubricant used to separate the journal and bearing is usually a mineral oil refined from petroleum, but vegetable oils, silicon oils, greases etc., may be used.

TYPES OF SLIDING CONTACT BEARINGS

The sliding contact bearings in which the sliding action is guided in a straight line and carrying radial loads, (as shown in Fig.) may be called slipper or guide bearings. Such type of bearings are usually found in cross-head of steam engine.

The sliding contact bearings in which the sliding action is along the circumference of a circle or an arc of a circle and carrying radial loads are known as journal or sleeve bearings.

When the angle of contact of the bearing with the journal is 360° then the bearing is called a full journal bearing. This type of bearing is commonly used in industrial machinery to accommodate bearing loads in any radial direction.



PROPERTIES OF SLIDING CONTACT BEARING MATERIALS

When the journal and the bearings are having proper lubrication i.e. there is a film of clean, non-corrosive lubricant in between, separating the two surfaces in contact, the only requirement of the bearing material is that they should have sufficient strength and rigidity. However, the conditions under which bearings must operate in service are generally far from ideal and thus the other properties as discussed below must be considered in selecting the best material.

1. **Compressive strength**. The maximum bearing pressure is considerably greater than the average pressure obtained by dividing the load to the projected area. Therefore the bearing material should have high compressive strength to withstand this maximum pressure so as to prevent extrusion or other permanent deformation of the bearing.
2. **Fatigue strength**. The bearing material should have sufficient fatigue strength so that it can withstand repeated loads without developing surface fatigue cracks. It is of major importance in aircraft and automotive engines.

3.Comformability. It is the ability of the bearing material to accommodate shaft deflections and bearing inaccuracies by plastic deformation (or creep) without excessive wear and heating.

4.Embeddability. It is the ability of bearing material to accommodate (or embed) small particles of dust, grit etc., without scoring the material of the journal.

5.Bondability. Many high capacity bearings are made by bonding one or more thin layers of a bearing material to a high strength steel shell. Thus, the strength of the bond i.e. bondability is an important consideration in selecting bearing material.

6.Corrosion resistance. The bearing material should not corrode away under the action of lubricating oil. This property is of particular importance in internal combustion engines where the same oil is used to lubricate the cylinder walls and bearings. In the cylinder, the lubricating oil comes into contact with hot cylinder walls and may oxidise and collect carbon deposits from the walls.

7.Thermal conductivity. The bearing material should be of high thermal conductivity so as to permit the rapid removal of the heat generated by friction.

8. **Thermal expansion**. The bearing material should be of low coefficient of thermal expansion,

3.Cast iron.

The cast iron bearings are usually used with steel journals. Such type of bearings are fairly successful where lubrication is adequate and the pressure is limited to 3.5 N/mm2 and speed to 40 metres per minute.

4.Silver.

The silver and silver lead bearings are mostly used in aircraft engines where the fatigue strength is the most important consideration.

5.Non-metallic bearings.

The various non-metallic bearings are made of carbon-graphite, rubber, wood and plastics. The carbon-graphite bearings are self lubricating, dimensionally stable over a wide range of operating conditions, chemically inert and can operate at higher temperatures than other bearings.

PROPERTIES OF LUBRICANTS

1. Viscosity. It is the measure of degree of fluidity of a liquid. It is a physical property by virtue of which an oil is able to form, retain and offer resistance to shearing a buffer film- under heat and pressure. The greater the heat and pressure, the greater viscosity is required of a lubricant to prevent thinning and squeezing out of the film. The motion is accompanied by a linear slip or shear between the particles throughout the entire height (h) of the film thickness. If A is the area of the plate in contact with the lubricant, then the unit shear stress is given by $\tau = P/A$

2 **Oiliness.** It is a joint property of the lubricant and the bearing surfaces in contact. It is a measure of the lubricating qualities under boundary conditions where base metal to metal is prevented only by absorbed film. There is no absolute measure of oiliness.

3. **Density.** This property has no relation to lubricating value but is useful in changing the kinematic viscosity to absolute viscosity. Mathematically Absolute viscosity = $\rho \times$ Kinematic viscosity (in m2/s) where ρ = Density of the lubricating oil. The density of most of the oils at 15.5°C varies from 860 to 950 kg / m3 (the average value may be taken as 900 kg / m3). The density at any other temperature (t) may be obtained from the following relation, i.e. $\rho t = \rho 15.5 - 0.000\,657\, t$ where $\rho 15.5$ = Density of oil at 15.5° C.

4. **Viscosity index.** The term viscosity index is used to denote the degree of variation of viscosity with temperature.

5. **Flash point**. It is the lowest temperature at which an oil gives off sufficient vapour to support a momentary flash without actually setting fire to the oil when a flame is brought within 6 mm at the surface of the oil.

6. **Fire point**. It is the temperature at which an oil gives off sufficient vapour to burn it continuously when ignited.

7. **Pour point or freezing point.** It is the temperature at which an oil will cease to flow when cooled.

TERMS USED IN HYDRODYNAMIC JOURNAL BEARING

bearing is shown in Fig. in which O is the centre of the journal and O′ is the centre of the bearing.

A hydrodynamic journal

Let, D = Diameter of the bearing,

d = Diameter of the journal, and l = Length of the bearing.

The following terms used in hydrodynamic journal bearing are important from the subject point of view :

1. Diametral clearance. It the difference between the diameters of the bearing and the journal. Mathematically, diametral clearance, c = D – d

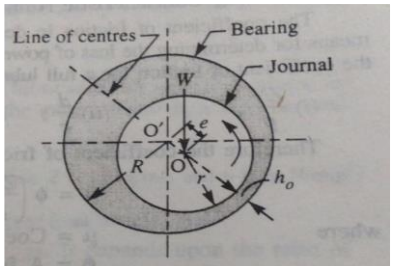


Fig. 26.7. Hydrodynamic journal bearing.

Note : The diametral clearance (c) in a bearing should be small enough to produce the necessary velocity gradient, so that the pressure built up will support the load. Also the small clearance has the advantage of decreasing side leakage. A commonly used clearance in industrial machines is 0.025 mm per cm of journal diameter.

2. Radial clearance. It is the difference between the radii of the bearing and the journal. Mathematically, radial clearance,

$$c1 = (D-d)/2 = R - r = c/2$$

3. Diametral clearance ratio. It is the ratio of the diametral clearance to the diameter of the journal. Mathematically, diametral clearance ratio

$$= c/d = (D- d)/d$$

4.Eccentricity. It is the radial distance between the centre (O) of the bearing and the displaced centre (O′) of the bearing under load. It is denoted by e.

5.Minimum oil film thickness. It is the minimum distance between the bearing and the journal,

under complete lubrication condition. It is denoted by h0 and occurs at the line of centres as shown in fig. Its value may be assumed as c / 4.

6.Attitude or eccentricity ratio. It is the ratio of the eccentricity to the radial clearance. Mathematically, attitude or eccentricity ratio, $\varepsilon = e/c1 = 1-ho/c1$

7. Short and long bearing. If the ratio of the length to the diameter of the journal (i.e. l / d) is less than 1, then the bearing is said to be short bearing.

On the other hand, if l/d is greater than 1, then the bearing is known as long bearing

BEARING CHARACTERISTIC NUMBER AND BEARING MODULUS FOR JOURNAL BEARINGS

The coefficient of friction in design of bearings is of great importance, because it affords a means for determining the loss of power due to bearing friction. It has been shown by experiments that the coefficient of friction for a full lubricated journal bearing is a function of three variables, i.e. (i) ; ZN /p (ii) d/ c and (iii) l /d

Therefore the coefficient of friction may be expressed as

$$\mu = \varphi \left[ZN/p,\ d/c\ ,\ l/d \right]$$

where μ = Coefficient of friction,

φ = A functional relationship,

Z = Absolute viscosity of the lubricant, in kg / m-s, N = Speed of the journal in r.p.m.,

p = Bearing pressure on the projected bearing area in N/mm^2,

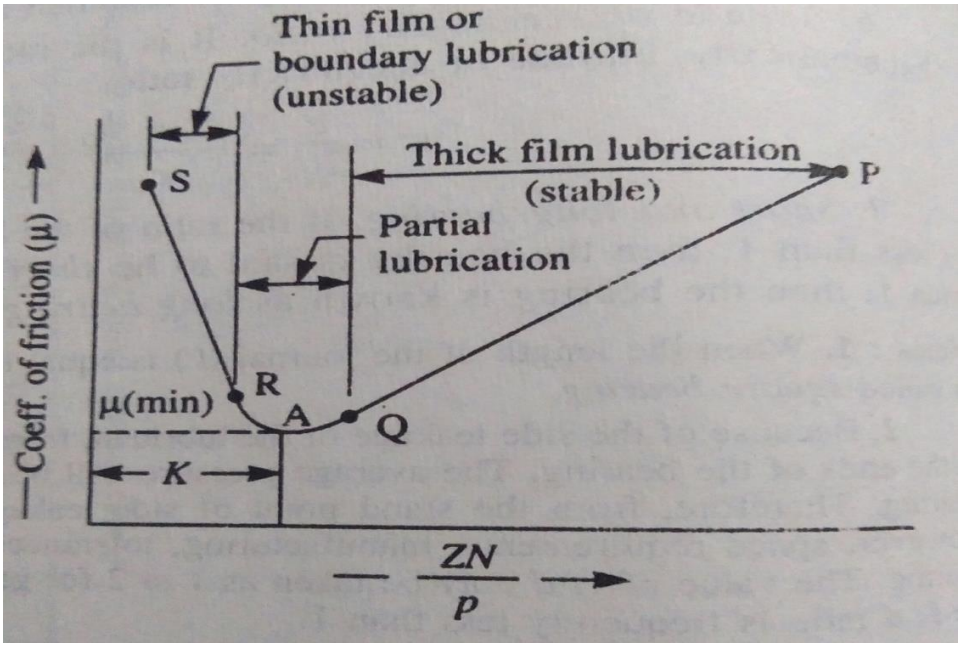
= Load on the journal ÷ l × d d

= Diameter of the journal,

l = Length of the bearing, and c

= Diametral clearance.

The factor ZN / p is termed as bearing characteristic number and is a dimensionless number. The variation of coefficient of friction with the operating values of bearing characteristic number (ZN / p) as obtained by McKee brothers (S.A. McKee and T.R. McKee) in an actual test of friction is shown in Fig. The factor ZN/p helps to predict the performance of a bearing.



COEFFICIENT OF FRICTION FOR JOURNAL BEARINGS

In order to determine the coefficient of friction for well lubricated full journal bearings, the following empirical relation established by McKee based on the experimental data, may be used. Coefficient of friction,

$\mu = (33/10^{8}) (ZN/p) (d/c) + k$ (when Z is in kg / m-s and p is in N / mm^2)

where Z, N, p, d and c have usual meanings as discussed in previous article, and

k = Factor to correct for end leakage. It depends upon the ratio of length to the diameter of the bearing (i.e. l/d).

= 0.002 for l/d ratios of 0.75 to 2.8.

The operating values of ZN / p should be compared with values given in Table of design values of journal bearing to ensure safe margin between operating conditions and the point of film breakdown.

CRITICAL PRESSURE OF THE JOURNAL BEARING

The pressure at which the oil film breaks down so that metal to metal contact begins, is known as critical pressure or the minimum operating pressure of the bearing. It may be obtained by the following empirical relation, i.e.Critical pressure or minimum operating pressure,

$$p = (ZN/P)\,(d/c)^2$$

SOMMERFELD NUMBER

The Sommerfeld number is also a dimensionless parameter used extensively in the design of journal bearings. Mathematically,

$$\text{Sommerfeld number} = (ZN/4.75 \times 10^6)\,(d/c)^2$$

For design purposes, its value is taken as follows : 14.3×10^6

(when Z is in kg / m-s and p is in N / mm2)

HEAT GENERATED IN A JOURNAL BEARING

The heat generated in a bearing is due to the fluid friction and friction of the parts having relative motion. Mathematically, heat generated in a bearing,

$$Qg = \mu.W.V \text{ N-m/s or J/s or watts} \quad ...(i)$$

where μ = Coefficient of friction,

W = Load on the bearing in N,

DESIGN PROCEDURE FOR JOURNAL BEARING

The following procedure may be adopted in designing journal bearings, when the bearing load, the diameter and the speed of the shaft are known.

1.Determine the bearing length by choosing a ratio of l/d from Table

2.Check the bearing pressure, p = W / l.d from Table for probable satisfactory value.

3.Assume a lubricant from Table 26.2 and its operating temperature (t 0). This temperature should be between 26.5°C and 60°C with 82°C as a maximum for high temperature installations

such as steam turbines.

4.Determine the operating value of ZN / p for the assumed bearing temperature and check this value with corresponding values in Table, to determine the possibility of maintaining fluid film operation.

5.Assume a clearance ratio c/d from Table

6.Determine the coefficient of friction (μ) by using the relation

7.Determine the heat generated by using the relation as discussed

8.Determine the heat dissipated by using the relation as discussed

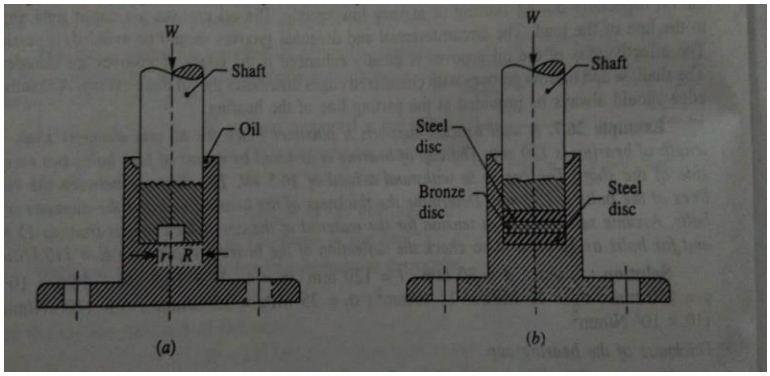
9.Determine the thermal equilibrium to see that the heat dissipated becomes atleast equal to the heat generated. In case the heat generated is more than the heat dissipated then either the bearing is redesigned or it is artificially cooled by water

THRUST BEARINGS

A thrust bearing is used to guide or support the shaft which is subjected to a load along the axis of the shaft. Such type of bearings are mainly used in turbines and propeller shafts. The thrust bearings are of the following two types :

1. Foot step or pivot bearings, and
2. Collar bearings.

In a foot step or pivot bearing, the loaded shaft is vertical and the end of the shaft rests within the bearing. In case of collar bearing, the shaft continues through the bearing. The shaft may be vertical or horizontal with single collar or many collars.



Footstep or Pivot Bearings

A simple type of footstep bearing, suitable for a slow running and lightly loaded shaft, is shown in Fig. If the shaft is not of steel, its end must be fitted with a steel face. The shaft is guided in a gunmetal bush, pressed into the pedestal and prevented from turning by means of a pin. Since the wear is proportional to the velocity of the rubbing surface, which increases with the distance from the axis (i.e. radius) of the bearing, therefore the wear will be different at different radii. It may be noted that the wear is maximum at the outer radius and zero at the centre. In order to compensate for end wear, the following two methods are employed.

1.The shaft is counter-bored at the end, as shown in Fig.

2.The shaft is supported on a pile of discs. It is usual practice to provide alternate discs of different materials such as steel and bronze, as shown in Fig. (b), so that the next disc comes into play, if one disc seizes due to improper lubrication. It may be noted that a footstep bearing is difficult to lubricate as the oil is being thrown outwards from the centre by centrifugal force. In designing,

it is assumed that the pressure is uniformly distributed throughout the bearing surface.

Let W = Load transmitted over the bearing

surface, R = Radius of the bearing surface

(or shaft),

A = Cross-sectional area of the bearing surface,

p = Bearing pressure per unit area of the bearing surface between rubbing

surfaces,

μ = Coefficient of friction, and

N = Speed of the shaft in r.p.m.

When the pressure in uniformly distributed over the bearing area, then

$$p = W/A = W/\pi R^2$$

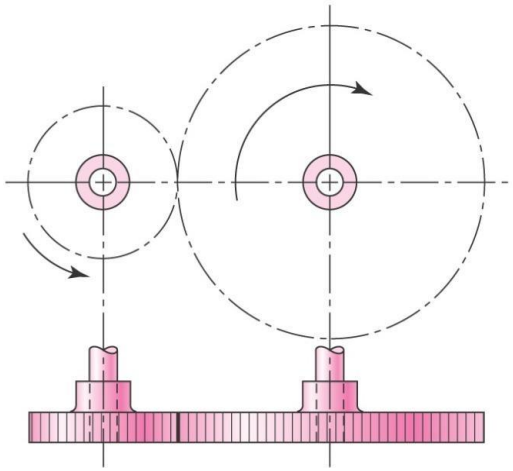
and the total frictional torque,

$$T = (2/3)\mu W R$$

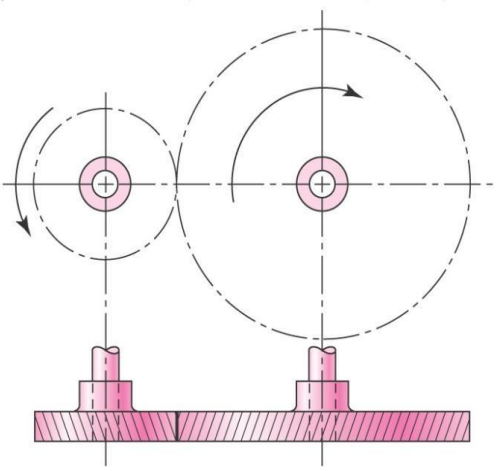
$\therefore$ Power lost in friction, $P = (2 \pi N T)/60$ ... (T being in N-m)

1.3 GEAR DESIGN

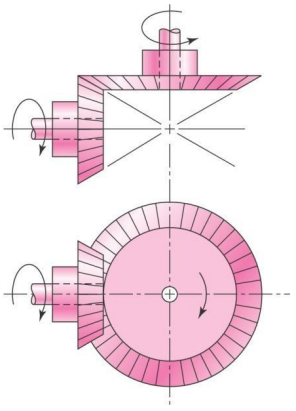
Types of Gears



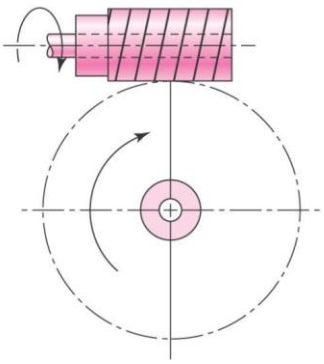
Spur



Helical

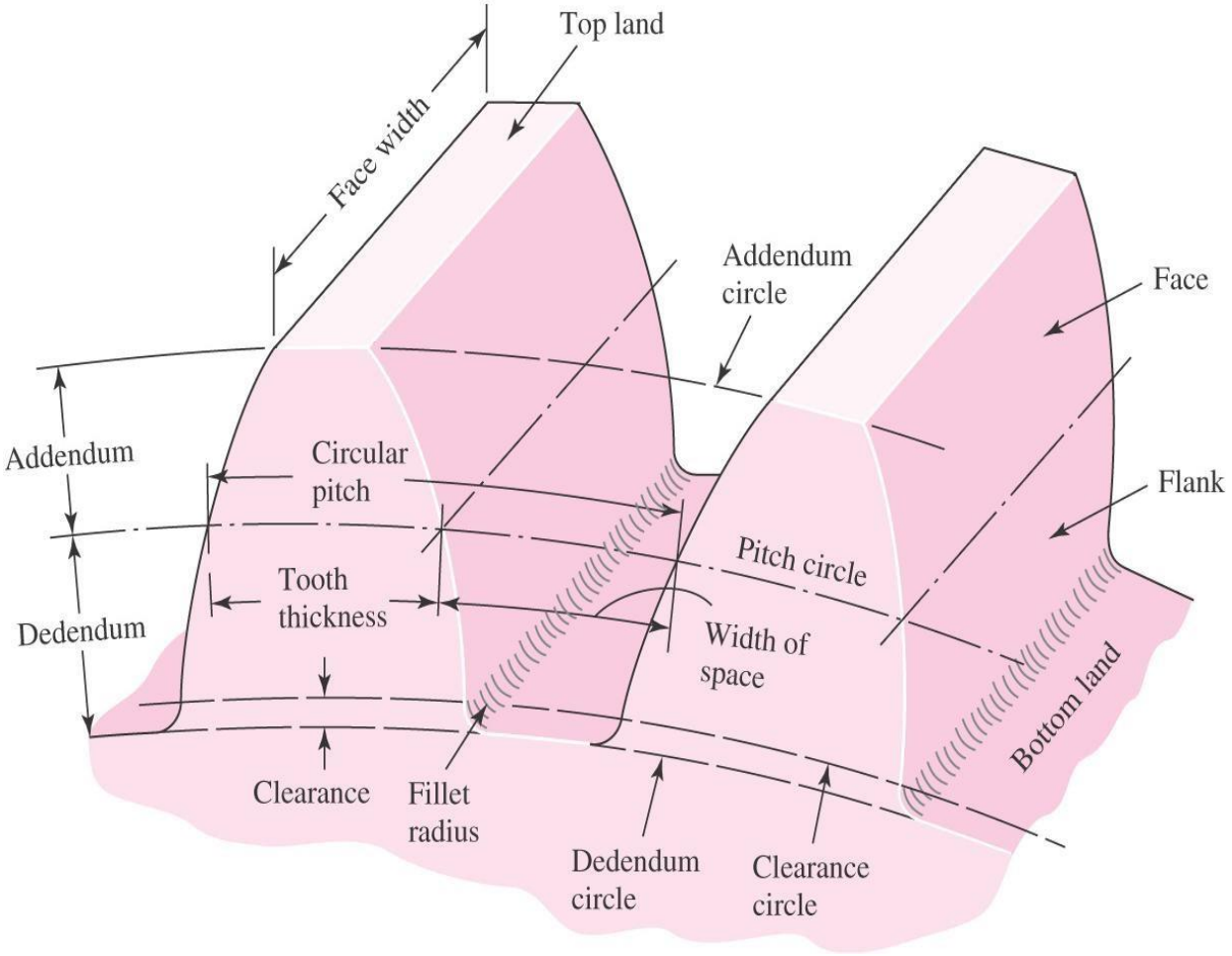


Bevel



Worm

Nomenclature of Spur-Gear Teeth



Tooth Size

$$P = \frac{N}{d} \tag{13–1}$$

$$m = \frac{d}{N} \tag{13–2}$$

$$p = \frac{\pi d}{N} = \pi m \tag{13–3}$$

$$pP = \pi \tag{13–4}$$

where P = diametral pitch, teeth per inch
N = number of teeth
d = pitch diameter, in
m = module, mm
d = pitch diameter, mm
p = circular pitch

Tooth Sizes in General Use

Diametral Pitch	
Coarse	2, $2\frac{1}{4}$, $2\frac{1}{2}$, 3, 4, 6, 8, 10, 12, 16
Fine	20, 24, 32, 40, 48, 64, 80, 96, 120, 150, 200

Modules	
Preferred	1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40, 50
Next Choice	1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14, 18, 22, 28, 36, 45

Standardized Tooth Systems (Spur Gears)

Tooth System	Pressure Angle ϕ, deg	Addendum a	Dedendum b
Full depth	20	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$
			$1.35/P_d$ or $1.35m$
	$22\frac{1}{2}$	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$
			$1.35/P_d$ or $1.35m$
	25	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$
			$1.35/P_d$ or $1.35m$
Stub	20	$0.8/P_d$ or $0.8m$	$1/P_d$ or $1m$

Table 13–1

Standardized Tooth Systems

- Common pressure angle ϕ : 20º and 25º
- Old pressure angle: 14 ½º
- Common face width:

$$3p < F < 5p$$

$$p = \frac{\pi}{P}$$

$$\frac{3\pi}{P} < F < \frac{5\pi}{P}$$

Conjugate Action

- When surfaces roll/slide against each other and produce constant angular velocity ratio, they are said to have *conjugate action.*
- Can be accomplished if instant center of velocity between the two bodies remains stationary between the grounded instant centers.

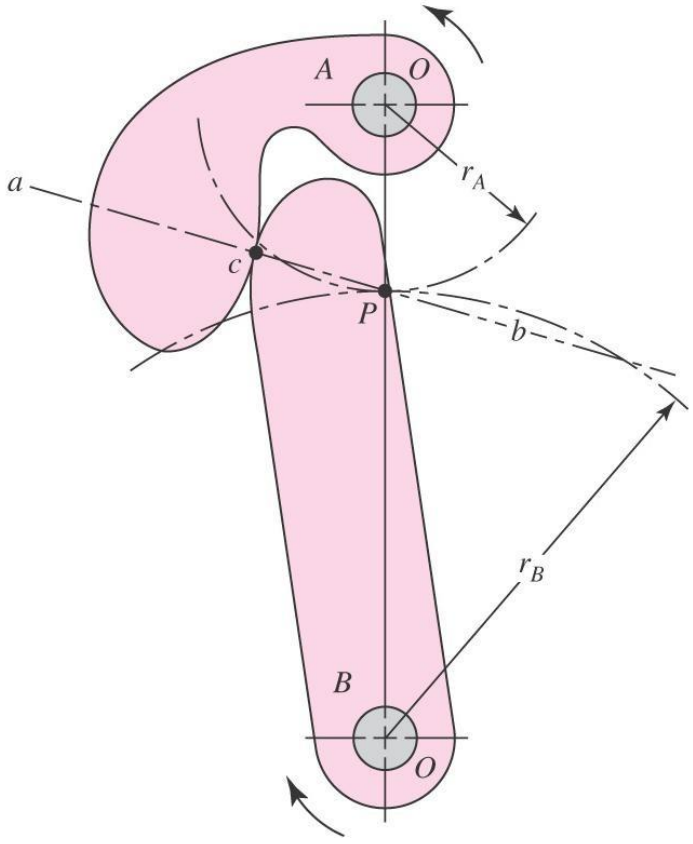


Fig. 13–6

- Forces are transmitted on *line of action* which is normal to the contacting surfaces.
- Angular velocity ratio is inversely proportional to the radii to point *P,* the *pitch point.*

$$\left|\frac{\omega_1}{\omega_2}\right| = \frac{r_2}{r_1} \qquad (13\text{–}5)$$

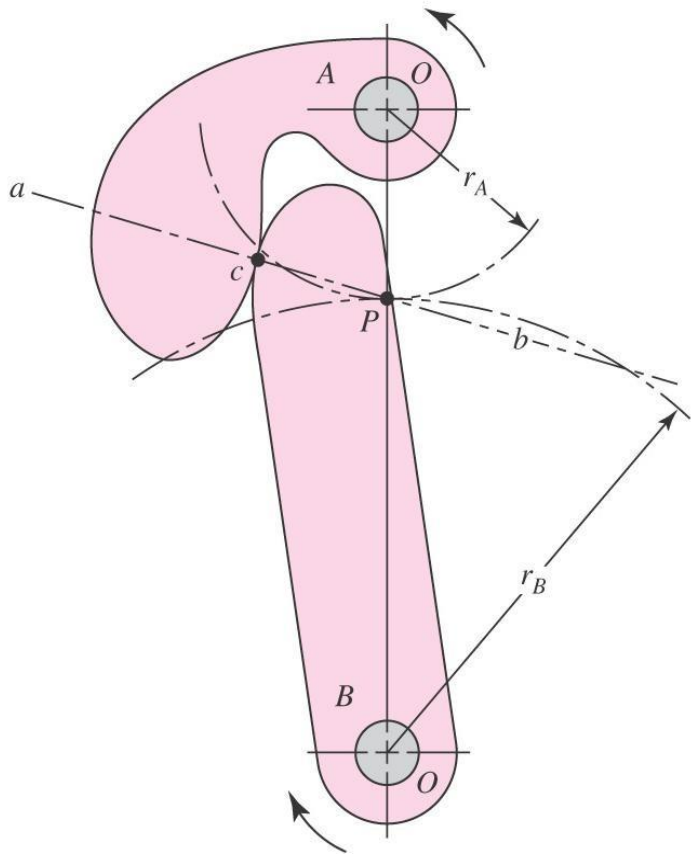


Fig. 13–6

- Circles drawn through *P* from each fixed pivot are *pitch circles,* each with a *pitch radius.*

Involute Profile

- The most common conjugate profile is the *involute* profile.
- Can be generated by unwrapping a string from a cylinder, keeping the string taut and tangent to the cylinder.
- Circle is called *base circle*.

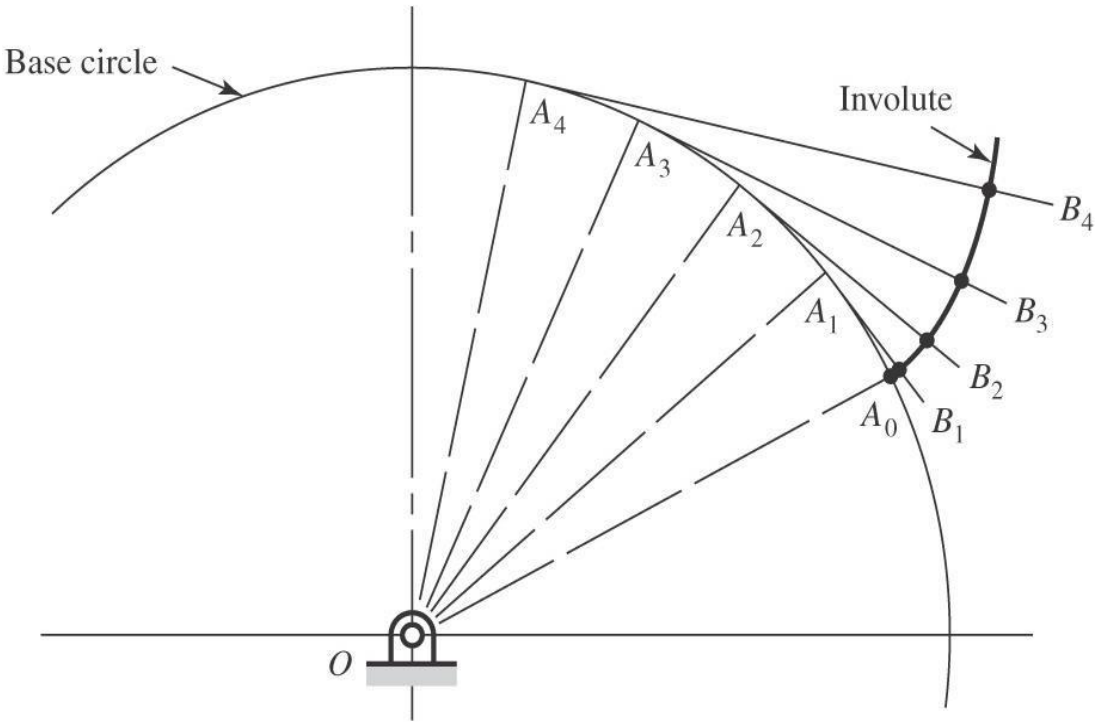


Fig. 13–8

Involute Profile Producing Conjugate Action

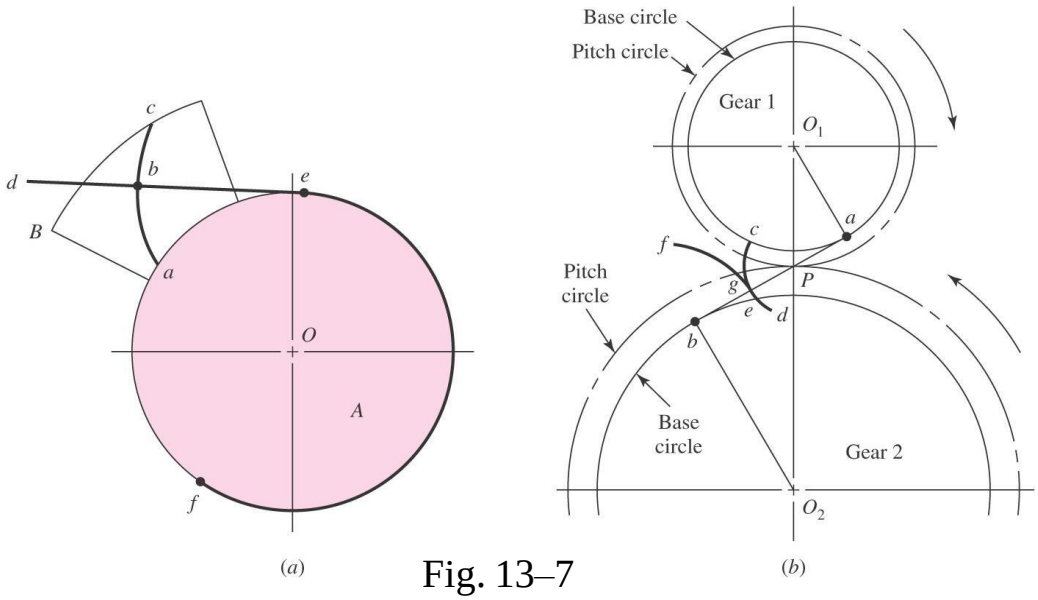


Fig. 13–7

Circles of a Gear Layout

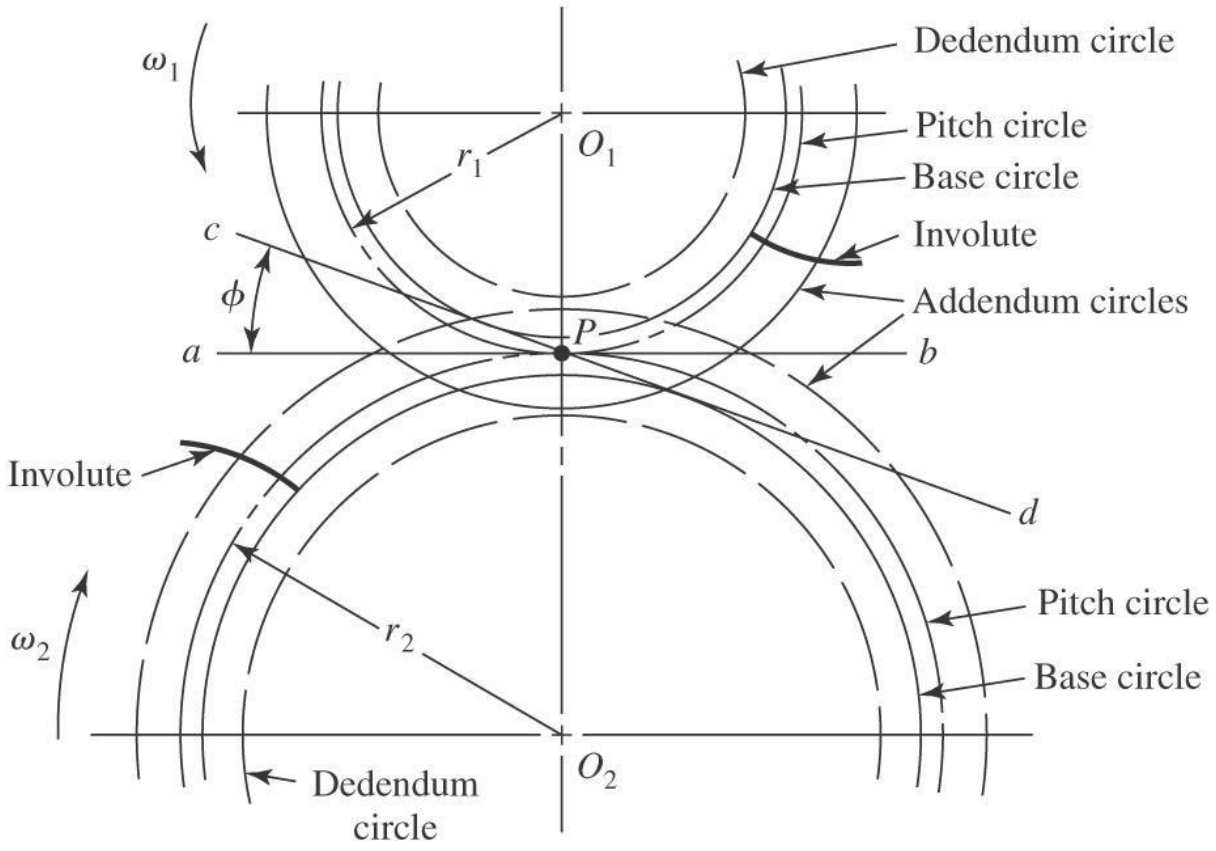


Fig. 13–9

Shigley's Mechanical Engineering Design

Sequence of Gear Layout

- Pitch circles in contact
- Pressure line at desired pressure angle
- Base circles tangent to pressure line
- Involute profile from base circle
- Cap teeth at addendum circle at 1/P from pitch circle
- Root of teeth at dedendum circle at 1.25/P from pitch circle
- Tooth spacing from circular pitch, $p = \pi / P$

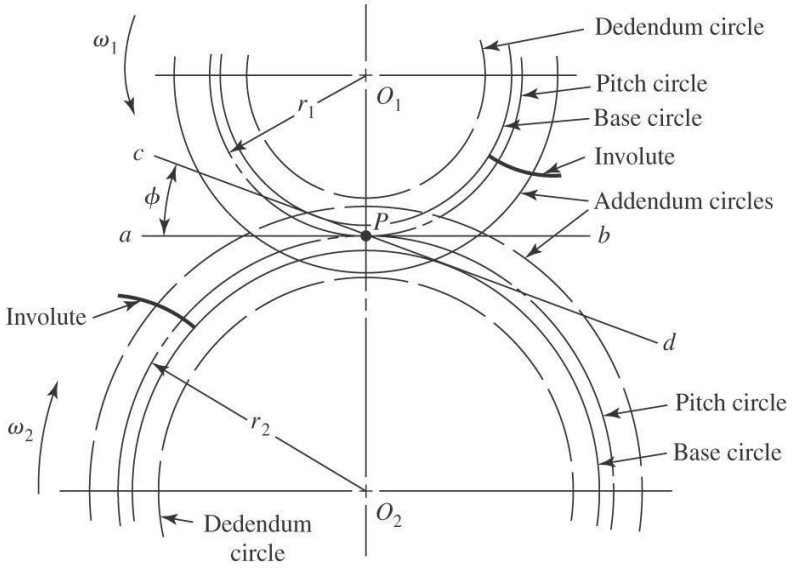


Fig. 13–9

Relation of Base Circle to Pressure Angle

$$r_b = r\cos\phi \qquad (13\text{–}6)$$

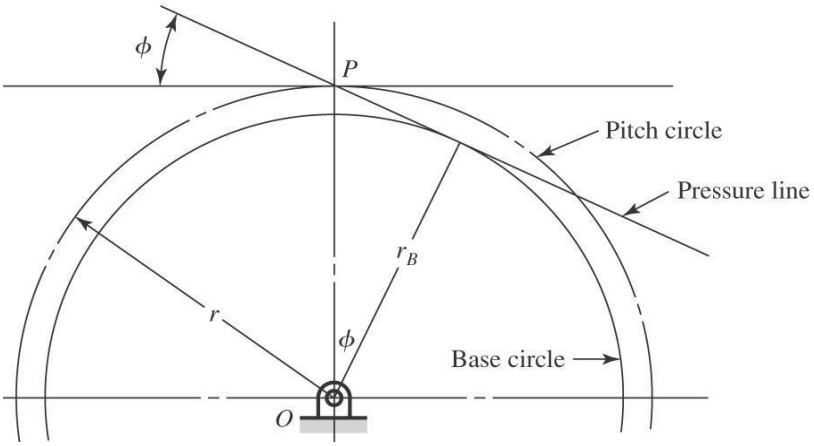


Fig. 13–10

Tooth Action

- First point of contact at *a* where flank of pinion touches tip of gear
- Last point of contact at *b* where tip of pinion touches flank of gear
- Line *ab* is *line of action*
- *Angle of action* is sum of *angle of approach* and *angle of recess*

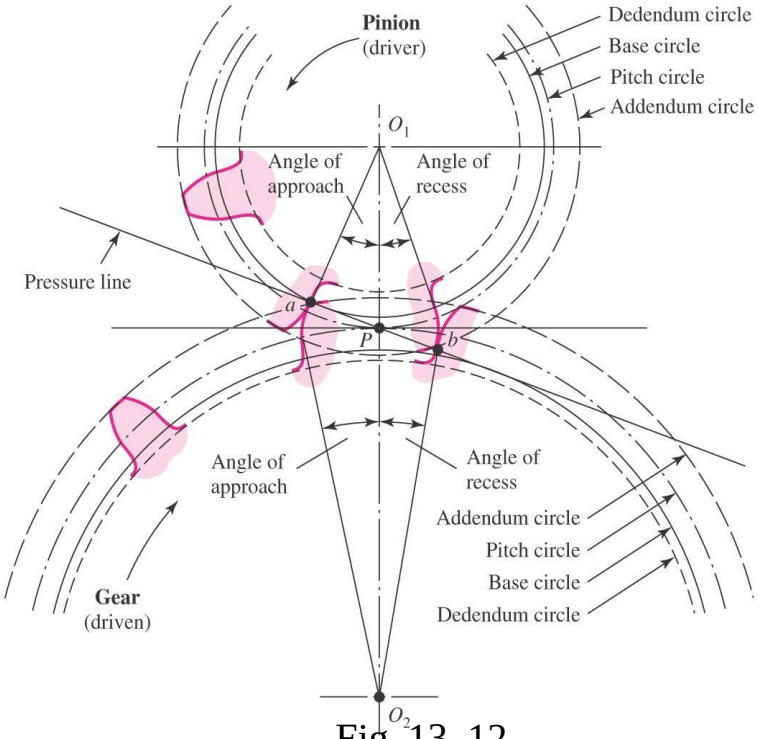


Fig. 13–12

Rack

- A *rack* is a spur gear with an pitch diameter of infinity.
- The sides of the teeth are straight lines making an angle to the line of centers equal to the pressure angle.
- The *base pitch* and *circular pitch*, shown in Fig. 13–13, are related by

$$p_b = p_c \cos\phi \qquad (13\text{–}7)$$

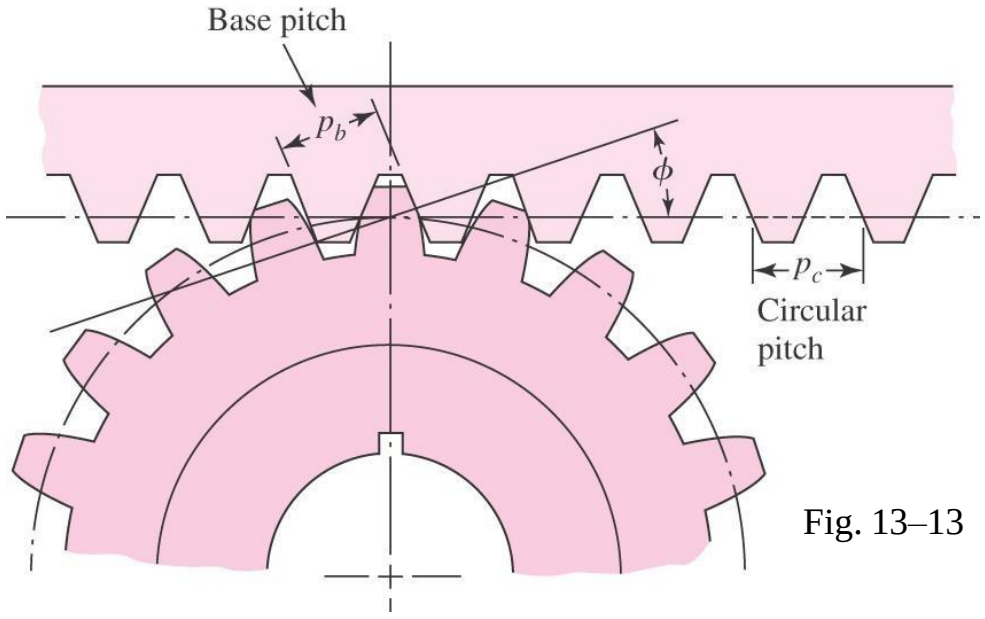


Fig. 13–13

Internal Gear

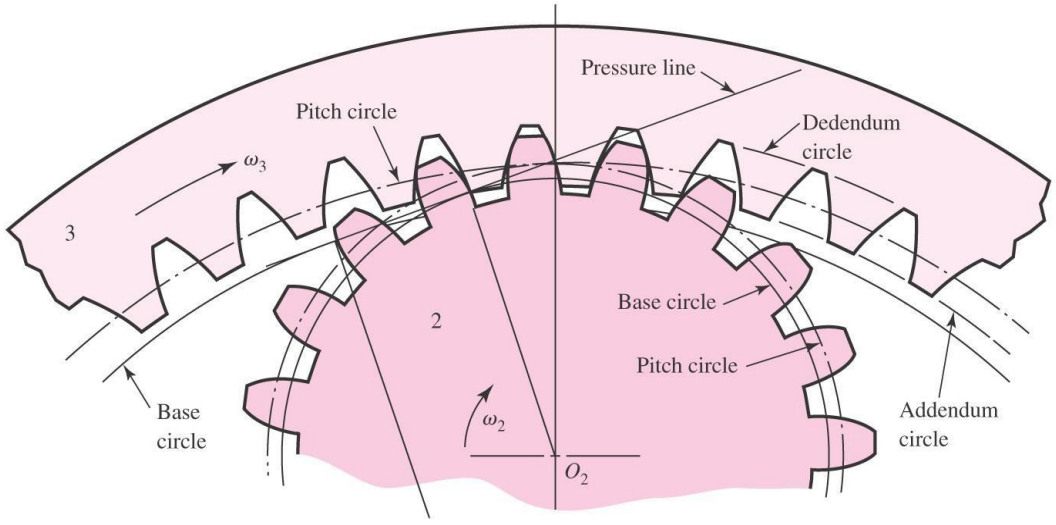


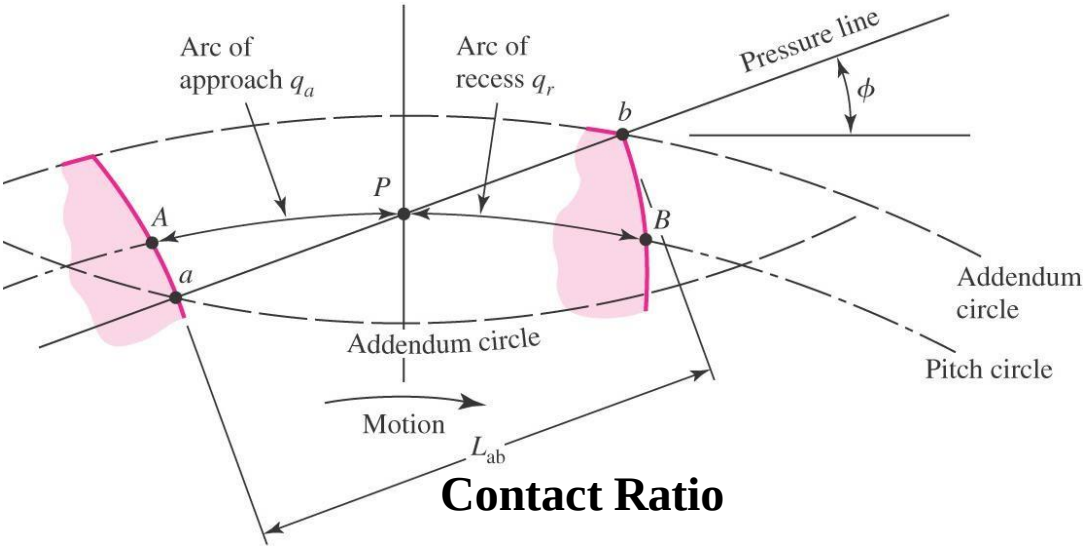
Fig. 13–14

Contact Ratio

- Arc of action q_t is the sum of the arc of approach q_a and the arc of recess q_r, that is $q_t = q_a + q_r$
- The *contact ratio* m_c is the ratio of the arc of action and the circular pitch.

$$m_c = \frac{q_t}{p} \qquad (13\text{–}8)$$

- The contact ratio is the average number of pairs of teeth in contact.



Contact Ratio

- Contact ratio can also be found from the length of the line of action

$$m_c = \frac{L_{ab}}{p \cos \phi} \qquad (13\text{–}9)$$

The contact ratio should be at least 1.2 ●

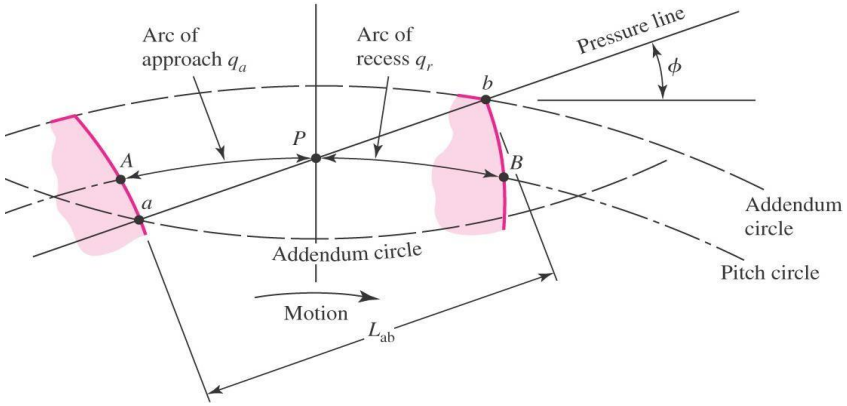


Fig. 13–15

Interference

- Contact of portions of tooth profiles that are not conjugate is called *interference*.
- Occurs when contact occurs below the base circle
- If teeth were produced by generating process (rather than stamping), then the generating process removes the interfering portion; known as *undercutting.*

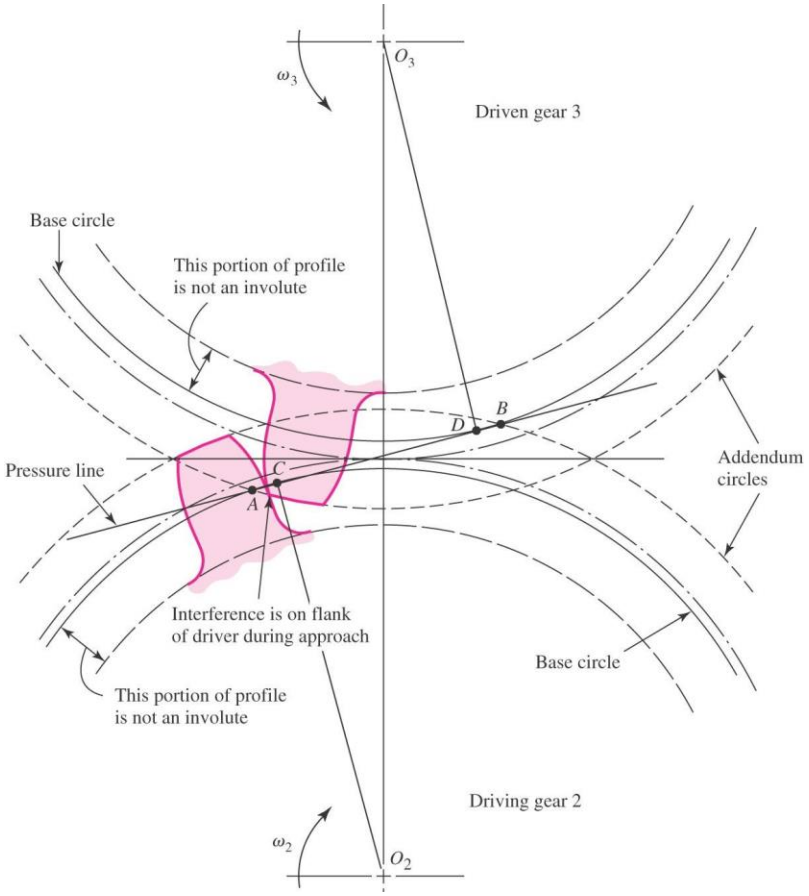


Fig. 13–16

Interference of Spur Gears

- On spur and gear with one-to-one gear ratio, smallest number of teeth which will not have interference is

$$N_P = \frac{2k}{3\sin^2\phi}\left(1+\sqrt{1+3\sin^2\phi}\right) \qquad (13\text{–}10)$$

- k =1 for full depth teeth. k =0.8 for stub teeth
- On spur meshed with larger gear with gear ratio $m_G = N_G/N_P = m$, the smallest number of teeth which will not have interference is

$$N_P = \frac{2k}{(1+2m)\sin^2\phi}\left(m+\sqrt{m^2+(1+2m)\sin^2\phi}\right) \qquad (13\text{–}11)$$

Interference of Spur Gears

- Largest gear with a specified pinion that is interference-free is

$$N_G = \frac{N_P^2\sin^2\phi - 4k^2}{4k - 2N_P\sin^2\phi} \qquad (13\text{–}12)$$

- Smallest spur pinion that is interference-free with a rack is

$$N_P = \frac{2(k)}{\sin^2\phi} \qquad (13\text{–}13)$$

Interference

- For 20º pressure angle, the most useful values from Eqs. (13–11) and (13–12) are calculated and shown in the table below.

Minimum N_P	Max N_G	Integer Max N_G	Max Gear Ratio $m_G = N_G/N_P$
13	16.45	16	1.23
14	26.12	26	1.86
15	45.49	45	3
16	101.07	101	6.31
17	1309.86	1309	77

Interference

- Increasing the pressure angle to 25º allows smaller numbers of teeth

Minimum N_P	Max N_G	Integer Max N_G	Max Gear Ratio $m_G = N_G/N_P$
9	13.33	13	1.44
10	32.39	32	3.2
11	249.23	249	22.64

Interference

- Interference can be eliminated by using more teeth on the pinion.
- However, if tooth size (that is diametral pitch P) is to be maintained, then an increase in teeth means an increase in diameter, since $P = N/d$.
- Interference can also be eliminated by using a larger pressure angle. This results in a smaller base circle, so more of the tooth profile is involute.
- This is the primary reason for larger pressure angle.
- Note that the disadvantage of a larger pressure angle is an increase in radial force for the same amount of transmitted force.

Straight Bevel Gears

- To transmit motion between intersecting shafts

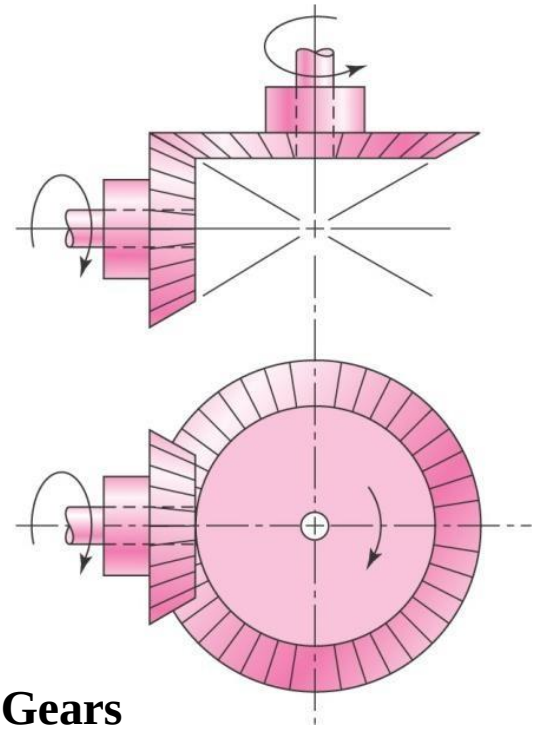


Fig. 13–3

Straight Bevel Gears

$$\tan\gamma = \frac{N_P}{N_G} \qquad \tan\Gamma = \frac{N_G}{N_P} \qquad (13\text{–}14)$$

- The shape of teeth, projected on back cone, is same as in a spur gear with radius r_b
- *Virtual number of teeth* in this virtual spur gear is

$$N' = \frac{2\pi r_b}{p} \qquad (13\text{–}15)$$

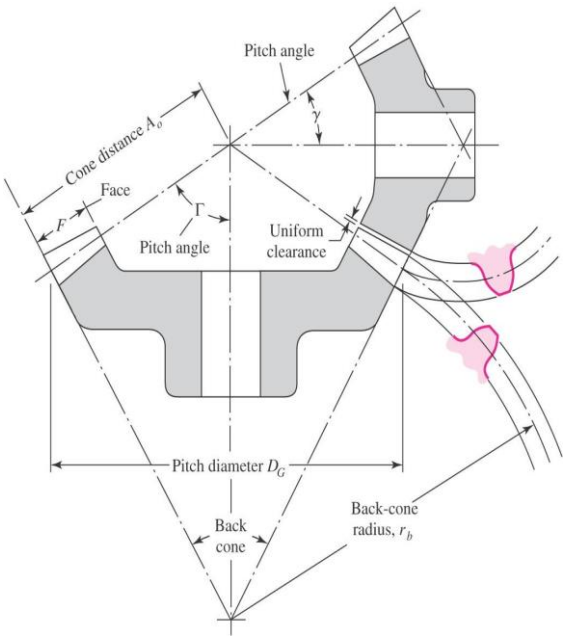


Fig. 13–20

Parallel Helical Gears

- Similar to spur gears, but with teeth makin a *helix angle* with respect to the gear centerline
- Adds axial force component to shaft and bearings
- Smoother transition of force between mating teeth due to gradual engagement and disengagement

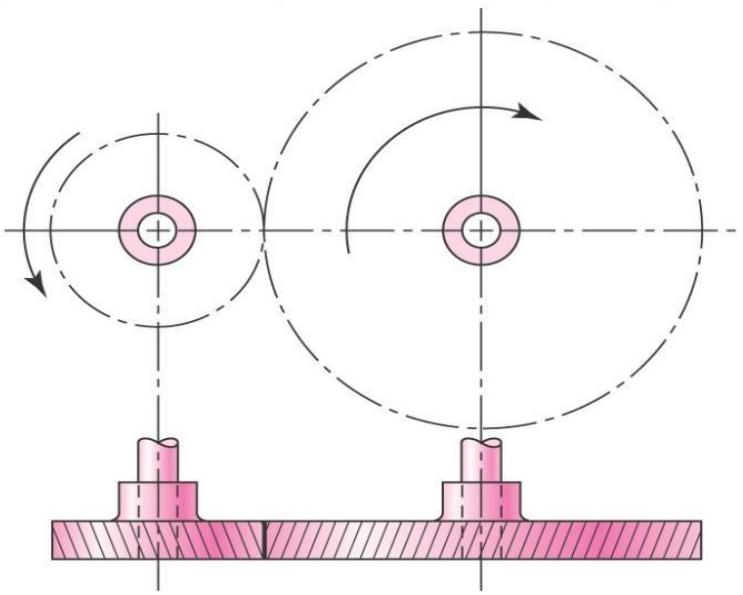


Fig. 13–2

Parallel Helical Gears

- Tooth shape is involute helicoid

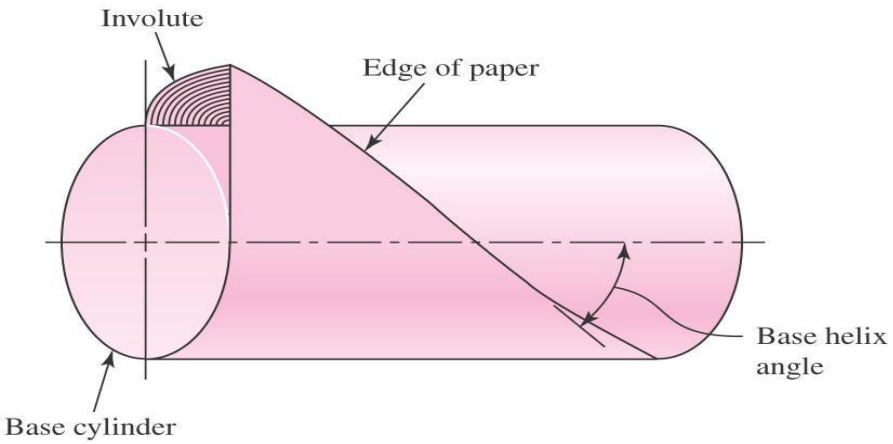


Fig. 13–21

Parallel Helical Gears

- *Transverse circular pitch* p_t is in the plane of rotation
- *Normal circular pitch* p_n is in the plane perpendicular to the teeth

$$p_n = p_t \cos \psi \qquad (13\text{–}16)$$

- *Axial pitch* p_x is along the direction of the shaft axis

$$p_x = \frac{p_t}{\tan \psi} \qquad (13\text{–}17)$$

- *Normal diametral pitch*

$$P_n = \frac{P_t}{\cos \psi} \qquad (13\text{–}18)$$

$$p_n P_n = \pi$$

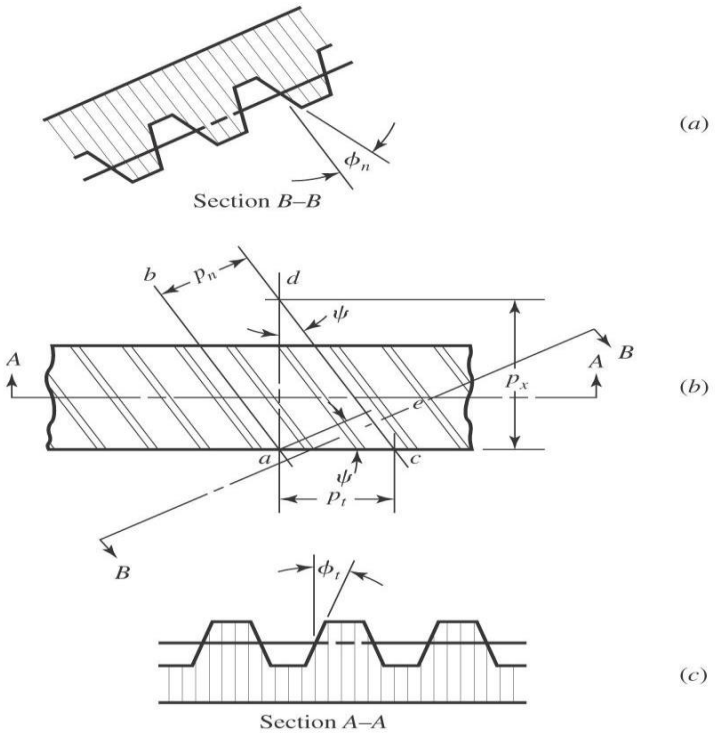


Fig. 13–22

Parallel Helical Gears

- Relationship between angles

$$\cos\psi = \frac{\tan\phi_n}{\tan\phi_t} \qquad (13\text{–}19)$$

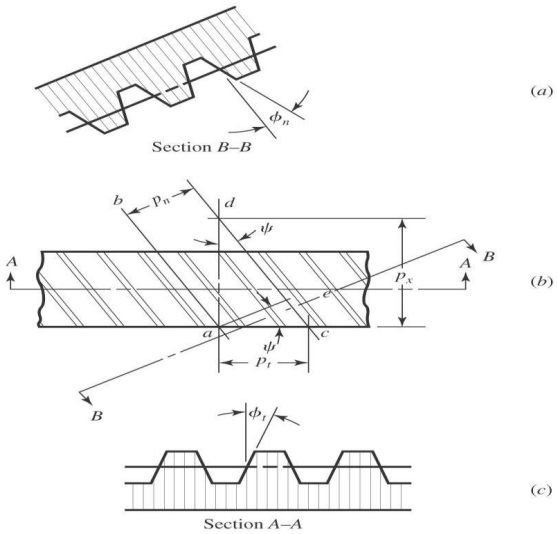


Fig. 13–22

Parallel Helical Gears

- Viewing along the teeth, the apparent pitch radius is greater than when viewed along the shaft.
- The greater virtual R has a greater *virtual number of teeth* N'

$$N' = \frac{N}{\cos^3\psi} \qquad (13\text{–}20)$$

- Allows fewer teeth on helical gears without undercutting.

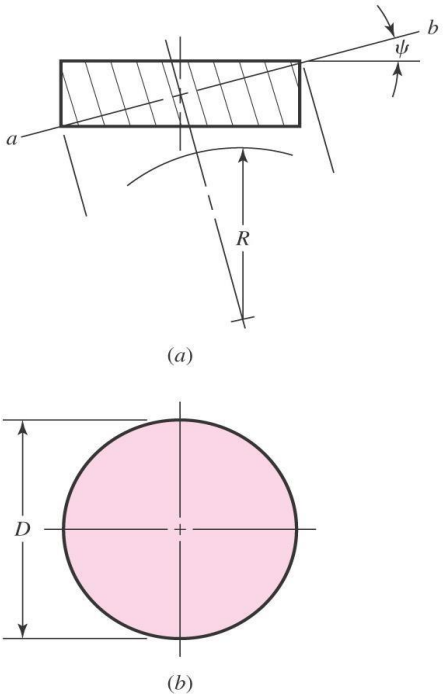


Fig. 13–23

Interference with Helical Gears

- On spur and gear with one-to-one gear ratio, smallest number of teeth which will not have interference is

$$N_P = \frac{2k\cos\psi}{3\sin^2\phi_t}\left(1+\sqrt{1+3\sin^2\phi_t}\right) \qquad (13\text{–}21)$$

- k =1 for full depth teeth. k =0.8 for stub teeth
- On spur meshed with larger gear with gear ratio $m_G = N_G/N_P = m$, the smallest number of teeth which will not have interference is

$$N_P = \frac{2k\cos\psi}{(1+2m)\sin^2\phi_t}\left[m+\sqrt{m^2+(1+2m)\sin^2\phi_t}\right] \qquad (13\text{–}22)$$

Interference with Helical Gears

- Largest gear with a specified pinion that is interference-free is

$$N_G = \frac{N_P^2\sin^2\phi_t - 4k^2\cos^2\psi}{4k\cos\psi - 2N_P\sin^2\phi_t} \qquad (13\text{–}23)$$

- Smallest spur pinion that is interference-free with a rack is

$$N_P = \frac{2k\cos\psi}{\sin^2\phi_t} \qquad (13\text{–}24)$$

Worm Gears

- Common to specify lead angle λ for worm and helix angle ψ_G for gear.
- Common to specify *axial pitch* p_x for worm and *transverse circular pitch* p_t for gear.
- Pitch diameter of gear is measured on plane containing worm axis

$$d_G = \frac{N_G p_t}{\pi} \qquad (13\text{–}25)$$

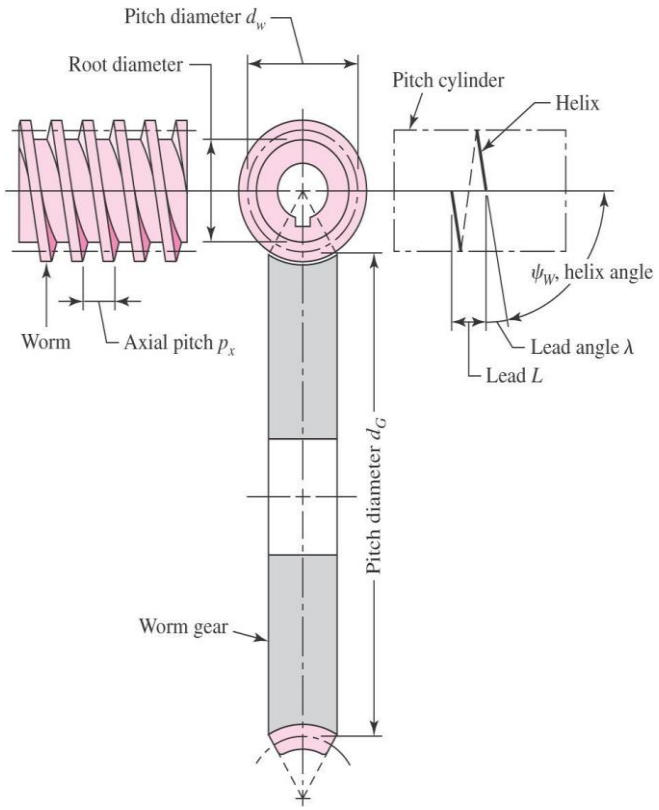


Fig. 13–24

Worm Gears

- Worm may have any pitch diameter.
- Should be same as hob used to cut the gear teeth
- Recommended range for worm pitch diameter as a function of center distance C,

$$\frac{C^{0.875}}{3.0} \le d_W \le \frac{C^{0.875}}{1.7} \qquad (13\text{–}26)$$

- Relation between *lead* L and *lead angle* λ,

$$L = p_x N_W \qquad (13\text{–}27)$$

$$\tan\lambda = \frac{L}{\pi d_W} \qquad (13\text{–}28)$$

Standard and Commonly Used Tooth Systems for Spur Gears

Tooth System	Pressure Angle ϕ, deg	Addendum a	Dedendum b
Full depth	20	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$
			$1.35/P_d$ or $1.35m$
	$22\frac{1}{2}$	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$
			$1.35/P_d$ or $1.35m$
	25	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$
			$1.35/P_d$ or $1.35m$
Stub	20	$0.8/P_d$ or $0.8m$	$1/P_d$ or $1m$

Table 13–1

Tooth Sizes in General Use

Diametral Pitch	
Coarse	$2, 2\frac{1}{4}, 2\frac{1}{2}, 3, 4, 6, 8, 10, 12, 16$
Fine	20, 24, 32, 40, 48, 64, 80, 96, 120, 150, 200

Modules	
Preferred	1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40, 50
Next Choice	1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14, 18, 22, 28, 36, 45

Table 13–2

Tooth Proportions for 20° Straight Bevel-

Item	Formula
Working depth	$h_k = 2.0/P$
Clearance	$c = (0.188/P) + 0.002$ in
Addendum of gear	$a_G = \frac{0.54}{P} + \frac{0.460}{P(m_{90})^2}$
Gear ratio	$m_G = N_G/N_P$
Equivalent 90° ratio	$m_{90} = m_G$ when $\Gamma = 90°$
	$m_{90} = \sqrt{m_G \frac{\cos\gamma}{\cos\Gamma}}$ when $\Gamma \neq 90°$
Face width	$F = 0.3A_0$ or $F = \frac{10}{P}$, whichever is smaller
Minimum number of teeth	Pinion | 16 15 14 13 Gear | 16 17 20 30

Standard Tooth Proportions for Helical Gears

Quantity	Formula	Quantity*	Formula
Addendum	$\frac{1.00}{P_n}$	External gears:	
Dedendum	$\frac{1.25}{P_n}$	Standard center distance	$\frac{D+d}{2}$
Pinion pitch diameter	$\frac{N_P}{P_n \cos\psi}$	Gear outside diameter	$D + 2a$
Gear pitch diameter	$\frac{N_G}{P_n \cos\psi}$	Pinion outside diameter	$d + 2a$
Normal arc tooth thickness†	$\frac{\pi}{P_n} - \frac{B_n}{2}$	Gear root diameter	$D - 2b$
Pinion base diameter	$d \cos\phi_t$	Pinion root diameter	$d - 2b$
		Internal gears:	
Gear base diameter	$D \cos\phi_t$	Center distance	$\frac{D-d}{2}$
Base helix angle	$\tan^{-1}(\tan\psi \cos\phi_t)$	Inside diameter	$D - 2a$
		Root diameter	$D + 2b$

Recommended Pressure Angles and Tooth Depths for Worm Gearing

Lead Angle λ, deg	Pressure Angle ϕ_n, deg	Addendum a	Dedendum b_G
0–15	$14\frac{1}{2}$	$0.3683p_x$	$0.3683p_x$
15–30	20	$0.3683p_x$	$0.3683p_x$
30–35	25	$0.2865p_x$	$0.3314p_x$
35–40	25	$0.2546p_x$	$0.2947p_x$
40–45	30	$0.2228p_x$	$0.2578p_x$

Table 13–5

Face Width of Worm Gear

- *Face width* F_G of a worm gear should be equal to the length of a tangent to the worm pitch circle between its points of intersection with the addendum circle

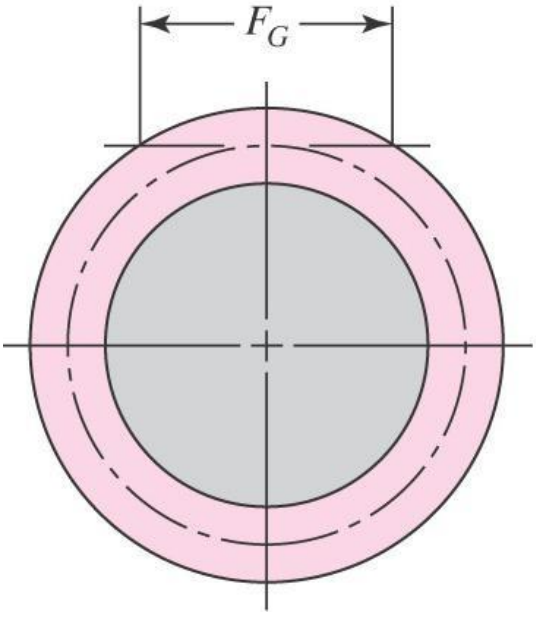


Fig. 13–25

Gear Trains

- For a pinion 2 driving a gear 3, the speed of the driven gear is

$$n_3 = \left|\frac{N_2}{N_3} n_2\right| = \left|\frac{d_2}{d_3} n_2\right| \qquad (13\text{–}29)$$

where n = revolutions or rev/min
N = number of teeth
d = pitch diameter

Relations for Crossed Helical Gears

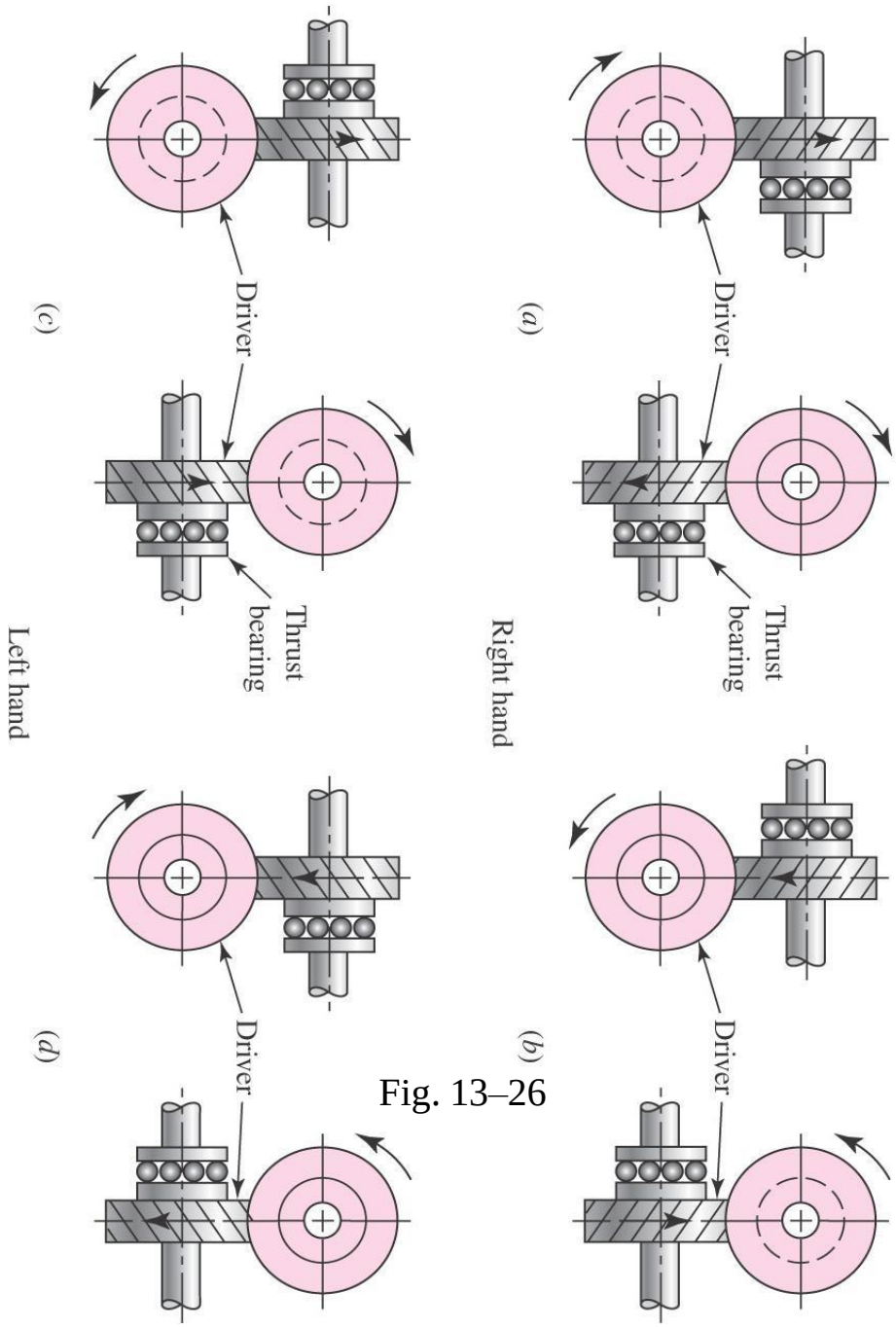


Fig. 13–26

Train Value

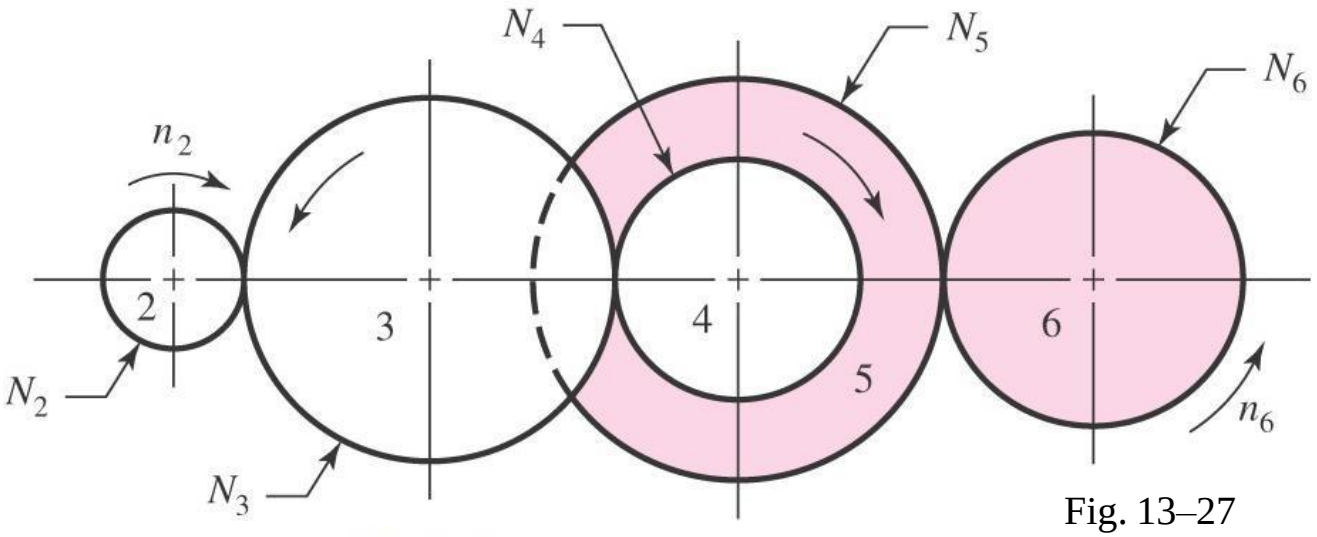


Fig. 13–27

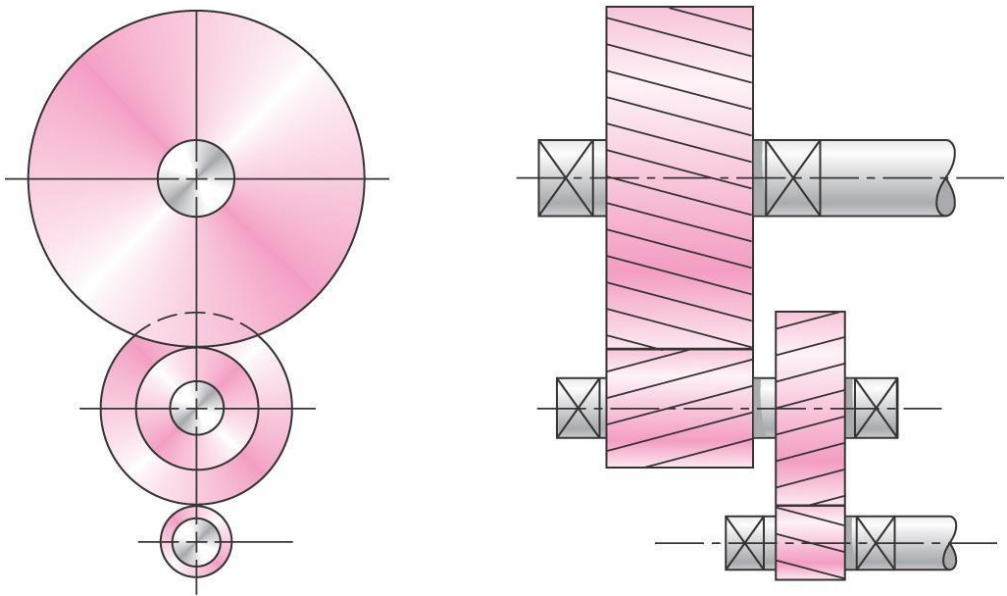
$$n_6 = -\frac{N_2}{N_3}\frac{N_3}{N_4}\frac{N_5}{N_6}n_2$$

$$e = \frac{\text{product of driving tooth numbers}}{\text{product of driven tooth numbers}} \qquad (13\text{–}30)$$

$$n_L = e n_F \qquad (13\text{–}31)$$

Compound Gear Train

- A practical limit on train value for one pair of gears is 10 to 1
- To obtain more, compound two gears onto the same shaft



Compound Reverted Gear Train

- A compound gear train with input and output shafts in-line
- Geometry condition must be satisfied

$$d_2/2 + d_3/2 = d_4/2 + d_5/2$$
$$P = N/d$$
$$N_2/(2P) + N_3/(2P) = N_4/(2P) + N_5/(2P)$$

$$N_2 + N_3 = N_4 + N_5$$

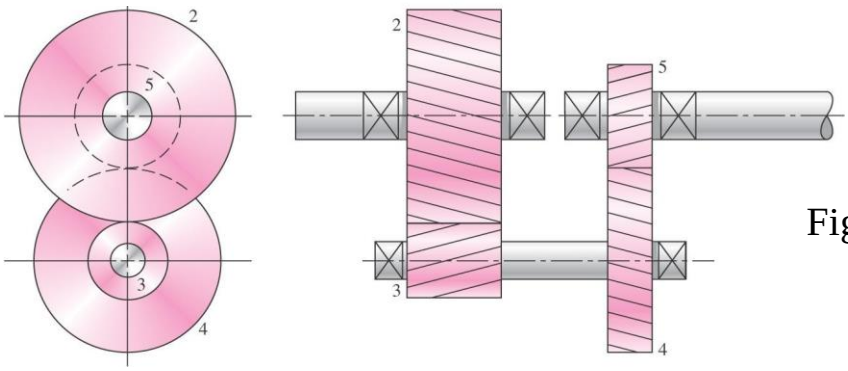


Fig. 13–29

Planetary Gear Train

- *Planetary,* or *epicyclic* gear trains allow the axis of some of the gears to move relative to the other axes
- *Sun gear* has fixed center axis
- *Planet gear* has moving center axis
- *Planet carrier* or *arm* carries planet axis relative to sun axis
- Allow for two degrees of freedom (i.e. two inputs)

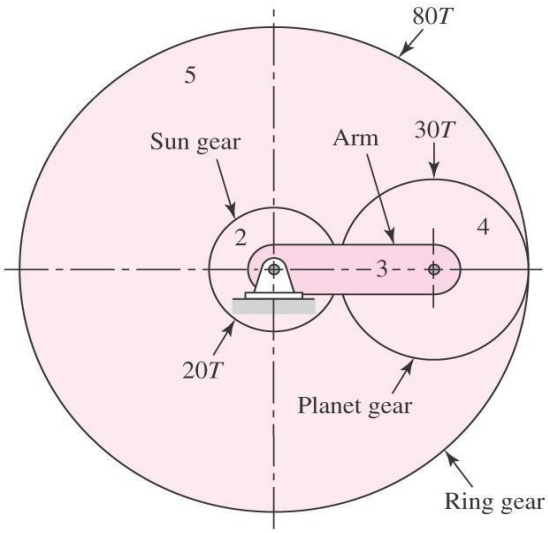


Fig. 13–30

Planetary Gear Trains

Train value is relative to arm

$$e = \frac{n_L - n_A}{n_F - n_A} \qquad (13\text{–}32)$$

where n_F = rev/min of first gear in planetary train

n_L = rev/min of last gear in planetary train

n_A = rev/min of arm

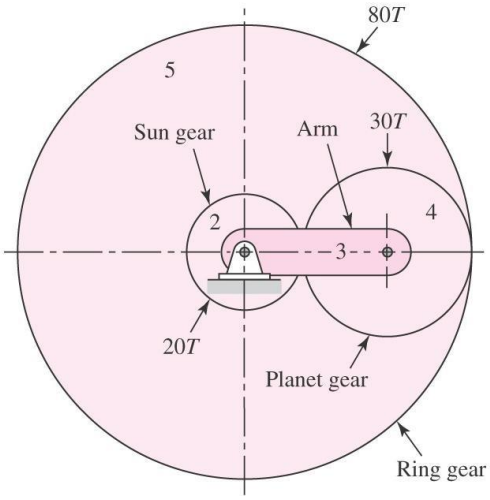


Fig. 13–30

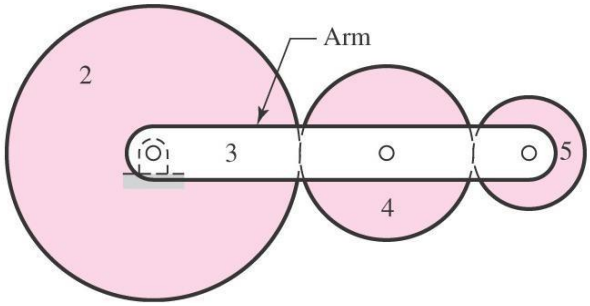
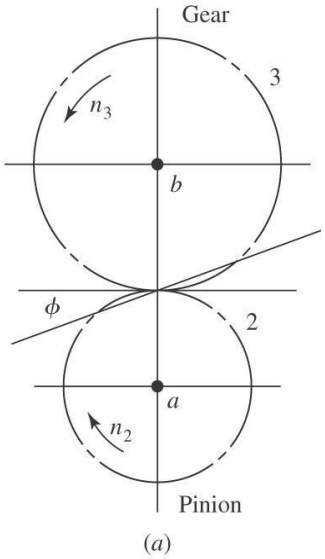


Fig. 13–31

Force Analysis – Spur Gearing



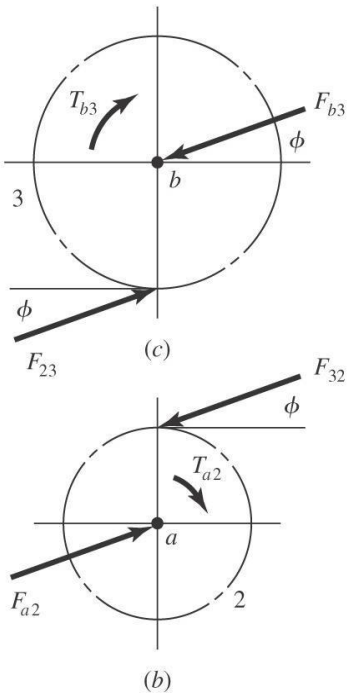


Fig. 13–32

Force Analysis – Spur Gearing

- *Transmitted load* W_t is the tangential load

$$W_t = F_{32}^t$$

- It is the useful component of force, transmitting the torque

$$T = \frac{d}{2} W_t$$

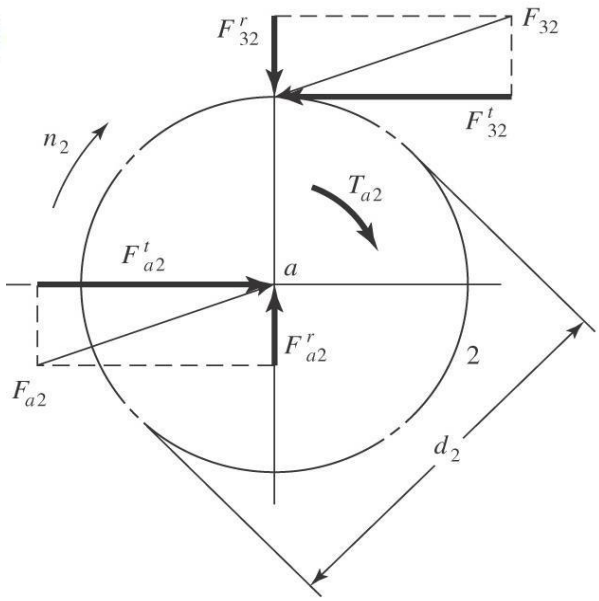


Fig. 13–33

Power in Spur Gearing

- *Transmitted power H*

$$H = T\omega = (W_t d/2)\,\omega \qquad (13\text{–}33)$$

- *Pitch-line velocity* is the linear velocity of a point on the gear at the radius of the pitch circle. It is a common term in tabulating gear data.

$$V = \pi d n/12 \qquad (13\text{–}34)$$

where V = pitch-line velocity, ft/min
d = gear diameter, in
n = gear speed, rev/min

Power in Spur Gearing

- Useful power relation in customary units,

$$W_t = 33\,000\frac{H}{V} \qquad (13\text{–}35)$$

where W_t = transmitted load, lbf
H = power, hp
V = pitch-line velocity, ft/min

- In SI units,

$$W_t = \frac{60\,000H}{\pi dn} \qquad (13\text{–}36)$$

where W_t = transmitted load, kN
H = power, kW
d = gear diameter, mm
n = speed, rev/min

Force Analysis – Bevel Gearing

$$W_t = \frac{T}{r_{av}} \qquad (13\text{–}37)$$

$$W_r = W_t \tan\phi \cos\gamma$$
$$W_a = W_t \tan\phi \sin\gamma \qquad (13\text{–}38)$$

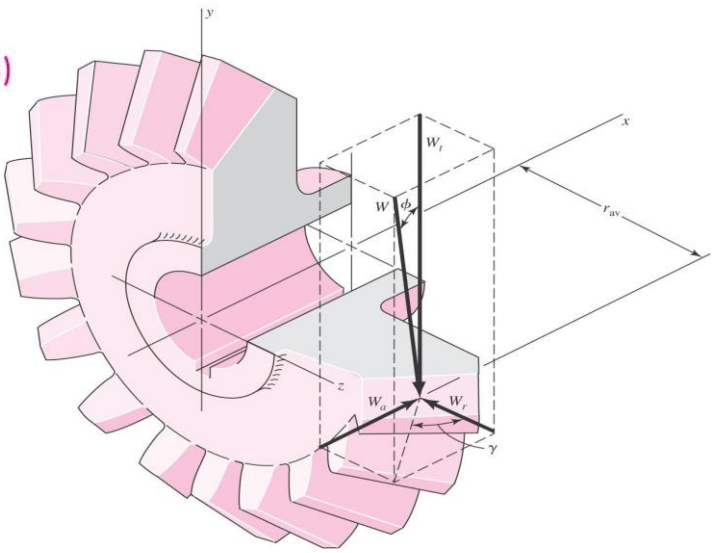
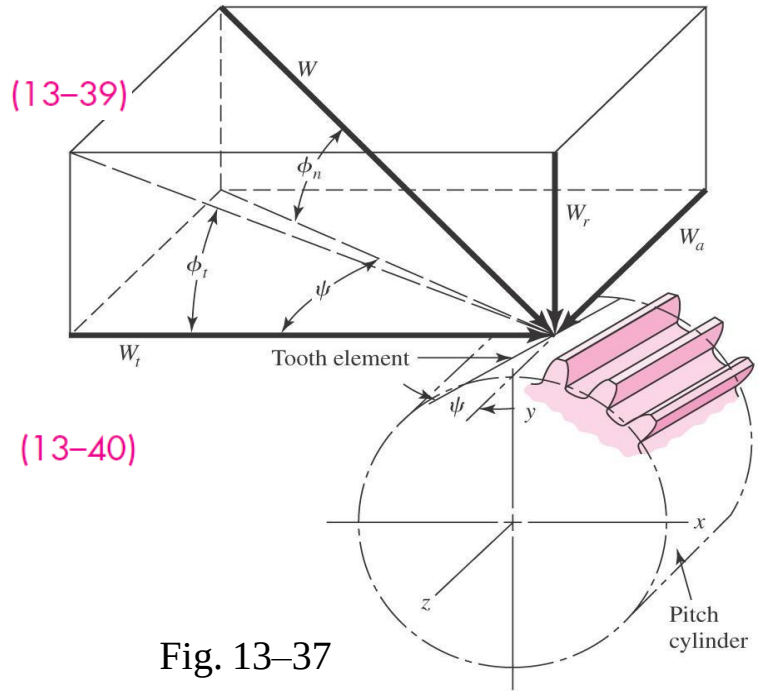


Fig. 13–35

Force Analysis – Helical Gearing

$$W_r = W \sin \phi_n$$
$$W_t = W \cos \phi_n \cos \psi$$
$$W_a = W \cos \phi_n \sin \psi \quad (13\text{–}39)$$



$$W_r = W_t \tan \phi_t$$
$$W_a = W_t \tan \psi \quad (13\text{–}40)$$
$$W = \frac{W_t}{\cos \phi_n \cos \psi}$$

Fig. 13–37

Force Analysis – Worm Gearing

$$W^x = W \cos \phi_n \sin \lambda$$
$$W^y = W \sin \phi_n \quad (13\text{–}41)$$
$$W^z = W \cos \phi_n \cos \lambda$$

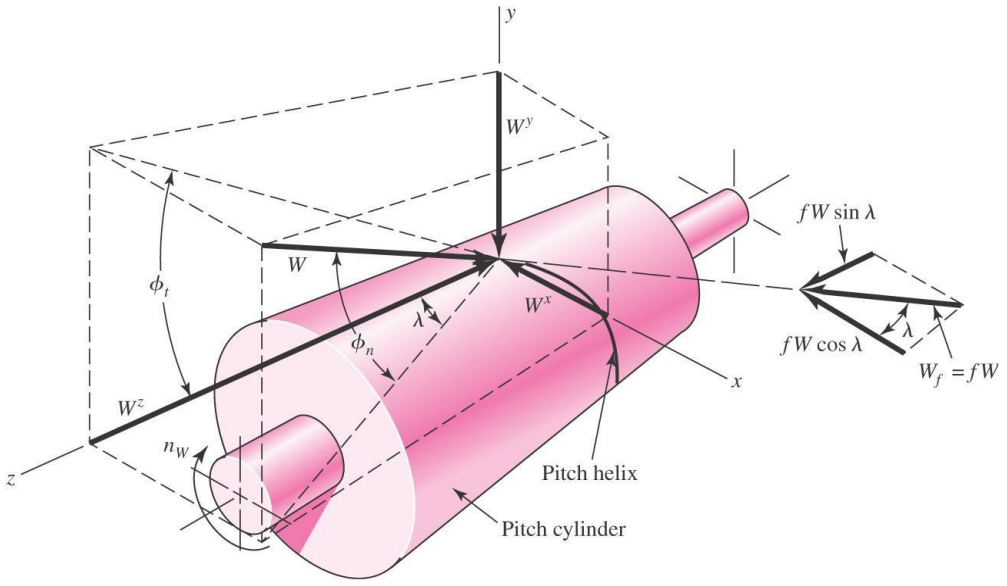


Fig. 13–40

Force Analysis – Worm Gearing

$$W_{Wt} = -W_{Ga} = W^x$$
$$W_{Wr} = -W_{Gr} = W^y \qquad (13\text{–}42)$$
$$W_{Wa} = -W_{Gt} = W^z$$

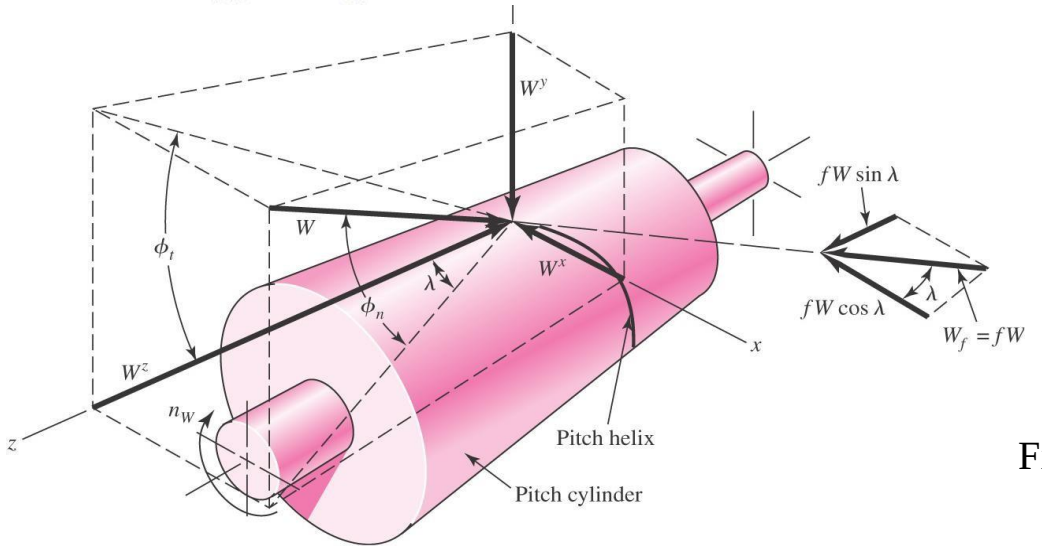


Fig. 13–40

Force Analysis – Worm Gearing

- Relative motion in worm gearing is sliding action
- Friction is much more significant than in other types of gears
- Including friction components, Eq. (13-41) can be expanded to

$$W^x = W(\cos\phi_n \sin\lambda + f\cos\lambda)$$
$$W^y = W\sin\phi_n \qquad (13\text{–}43)$$
$$W^z = W(\cos\phi_n \cos\lambda - f\sin\lambda)$$

- Combining with Eqs. (13-42) and (13-43),

$$W_f = fW = \frac{fW_{Gt}}{f\sin\lambda - \cos\phi_n\cos\lambda} \qquad (13\text{–}44)$$

$$W_{Wt} = W_{Gt}\frac{\cos\phi_n\sin\lambda + f\cos\lambda}{f\sin\lambda - \cos\phi_n\cos\lambda} \qquad (13\text{–}45)$$

Worm Gearing Efficiency

- *Efficiency* is defined as

$$\eta = \frac{W_{Wt}\,(\text{without friction})}{W_{Wt}\,(\text{with friction})}$$

- From Eq. (13–45) with $f = 0$ in the numerator,

$$\eta = \frac{\cos\phi_n - f\tan\lambda}{\cos\phi_n + f\cot\lambda} \qquad (13\text{–}46)$$

Worm Gearing Efficiency

- With typical value of $f = 0.05$, and $\phi_n = 20^\circ$, efficiency as a function of helix angle is given in the table.

Helix Angle ψ, deg	Efficiency η, %
1.0	25.2
2.5	45.7
5.0	62.0
7.5	71.3
10.0	76.6
15.0	82.7
20.0	85.9
30.0	89.1

Table 13–6

Worm Gearing Efficiency

- Coefficient of friction is dependent on relative or sliding velocity V_S
- V_G is pitch line velocity of gear
- V_W is pitch line velocity of worm

$$\mathbf{V}_W = \mathbf{V}_G + \mathbf{V}_S$$

$$V_S = \frac{V_W}{\cos\lambda} \qquad (13\text{–}47)$$

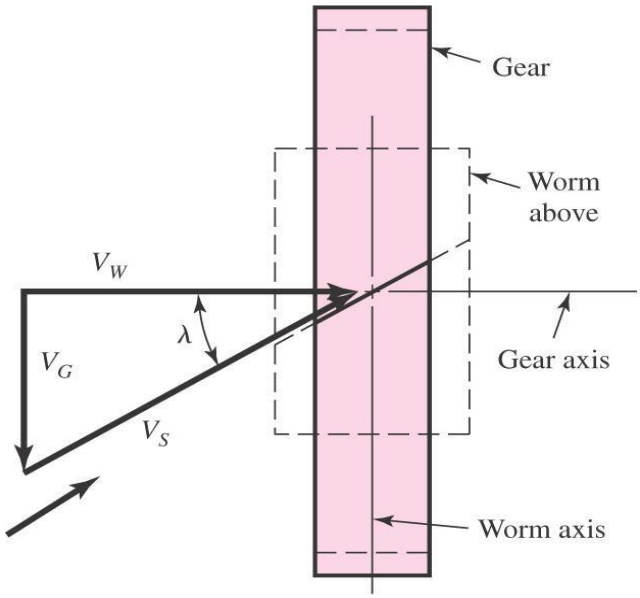


Fig. 13–41

Coefficient of Friction for Worm Gearing

- Graph shows representative values
- Curve A is for when more friction is expected, such as when gears are cast iron
- Curve B is for high-quality materials

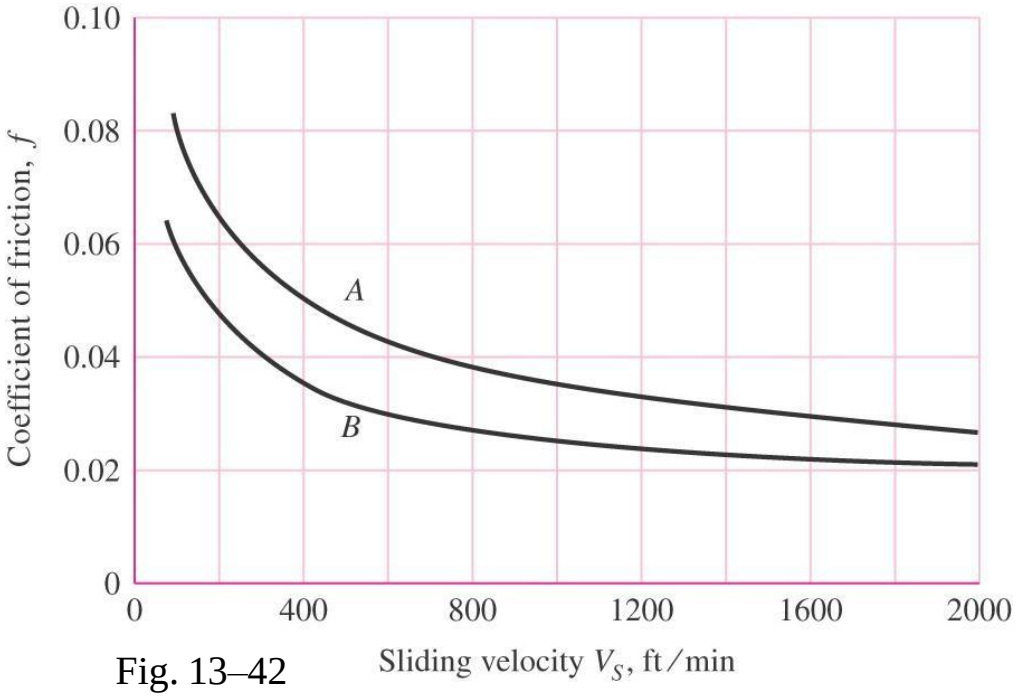
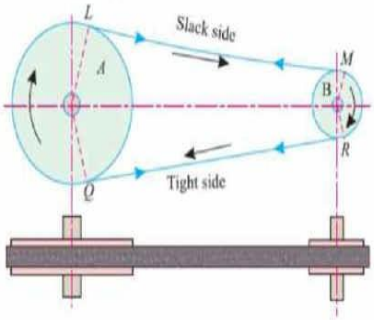


Fig. 13–42

DESIGN OF FLAT AND V BELT DRIVES

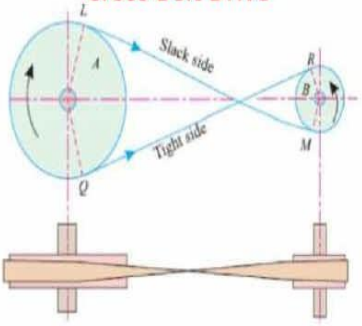
Types of Belt drives

Open Belt Drive



- two pulleys rotate in the same direction
- Length of the belt is smaller
- Angle of lap is different for driver and driven pulley

Cross Belt Drive



- pulleys rotate in the opposite directions
- Length of the belt is larger
- Angle of lap is same for driver and driven pulley

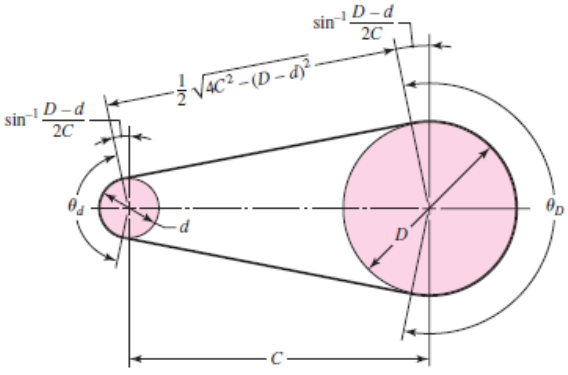
APPLICATIONS

- Electric Motors
- Machine Tools
- Automobiles
- Conveyors
- Alternators







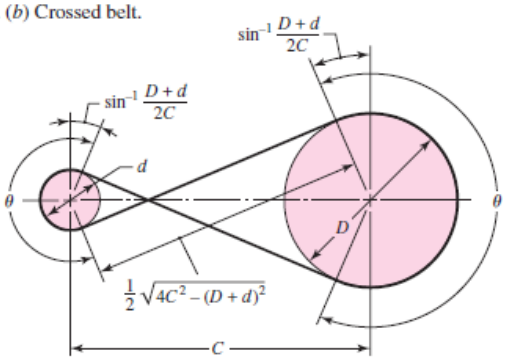


$$\theta_d = \pi - 2\sin^{-1}\frac{D-d}{2C}$$

$$\theta_D = \pi + 2\sin^{-1}\frac{D-d}{2C}$$

$$L = \sqrt{4C^2-(D-d)^2} + \tfrac{1}{2}(D\theta_D + d\theta_d)$$

(*a*)



$$\theta = \pi + 2\sin^{-1}\frac{D+d}{2C}$$

$$L = \sqrt{4C^2-(D+d)^2} + \tfrac{1}{2}(D+d)\theta$$

(*b*)

Flat-belt geometry. (*a*) Open belt. (*b*) Crossed belt.

Flat- and Round-Belt Drives

Modern flat-belt drives consist of a strong elastic core surrounded by an elastomer; these drives have distinct advantages over gear drives or V-belt drives. A flat-belt drive has an efficiency of about 98 percent, which is about the same as for a gear drive. On the other hand, the efficiency of a V-belt drive ranges from about 70 to 96 percent.[1] Flat-belt drives produce very little noise and absorb more torsional vibration from the system than either V-belt or gear drives.

When an open-belt drive (Fig. 17–1*a*) is used, the contact angles are found to be

$$\theta_d = \pi - 2\sin^{-1}\frac{D-d}{2C}$$

$$\theta_D = \pi + 2\sin^{-1}\frac{D-d}{2C} \tag{17–1}$$

where D = diameter of large pulley
d = diameter of small pulley
C = center distance
θ = angle of contact

The length of the belt is found by summing the two arc lengths with twice the distance between the beginning and end of contact. The result is

$$L = [4C^2 - (D-d)^2]^{1/2} + \frac{1}{2}(D\theta_D + d\theta_d) \tag{17–2}$$

A similar set of equations can be derived for the crossed belt of Fig. 17–2*b*. For this belt, the angle of wrap is the same for both pulleys and is

$$\theta = \pi + 2\sin^{-1}\frac{D+d}{2C} \tag{17–3}$$

The belt length for crossed belts is found to be

$$L = [4C^2 - (D+d)^2]^{1/2} + \frac{1}{2}(D+d)\theta \tag{17–4}$$

Now F_c is found as follows: with n being the rotational speed, in rev/min, of the pulley of diameter d, the belt speed is

$$V = \pi\, dn/12 \qquad \text{ft/min}$$

The weight w of a foot of belt is given in terms of the weight density γ in lbf/in^3 as $w = 12\gamma bt$ lbf/ft where b and t are in inches. F_c is written as

$$F_c = \frac{w}{g}\left(\frac{V}{60}\right)^2 = \frac{w}{32.17}\left(\frac{V}{60}\right)^2 \tag{e}$$

Figure 17–7 shows a free body of a pulley and part of the belt. The tight side tension F_1 and the loose side tension F_2 have the following additive components:

$$F_1 = F_i + F_c + \Delta F/2 = F_i + F_c + T/d \tag{f}$$

$$F_2 = F_i + F_c - \Delta F/2 = F_i + F_c - T/d \tag{g}$$

where F_i = initial tension
F_c = hoop tension due to centrifugal force
$\Delta F/2$ = tension due to the transmitted torque T
d = diameter of the pulley

The difference between F_1 and F_2 is related to the pulley torque. Subtracting Eq. (*g*) from Eq. (*f*) gives

$$F_1 - F_2 = \frac{2T}{d} \tag{h}$$

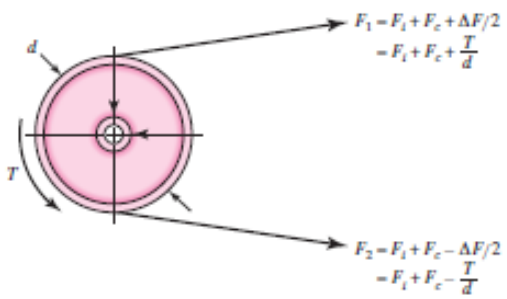
Adding Eqs. (*f*) and (*g*) gives

$$F_1 + F_2 = 2F_i + 2F_c$$

from which

$$F_i = \frac{F_1 + F_2}{2} - F_c \tag{i}$$

$F_1 + F_2 = 2F_i + 2F_c$



Forces and torques on a pulley.

The initial tension needs to be sufficient so that the difference between the F_1 and F_2 curve is $2T/d$. With no torque transmitted, the least possible belt tension is $F_1 = F_2 = F_c$.

The transmitted horsepower is given by

$$H = \frac{(F_1 - F_2)V}{33\,000} \tag{j}$$

$$F_1 = F_c + F_i \frac{2\exp(f\phi)}{\exp(f\phi) + 1} \tag{17–10}$$

$$F_2 = F_c + F_i \frac{2}{\exp(f\phi) + 1} \tag{17–11}$$

Ac

the nominal power as $H_d = H_{nom} K_s n_d$, where n_d is the design factor for exigencies. These effects are incorporated as follows:

$$(F_1)_a = b F_a C_p C_v \tag{17–12}$$

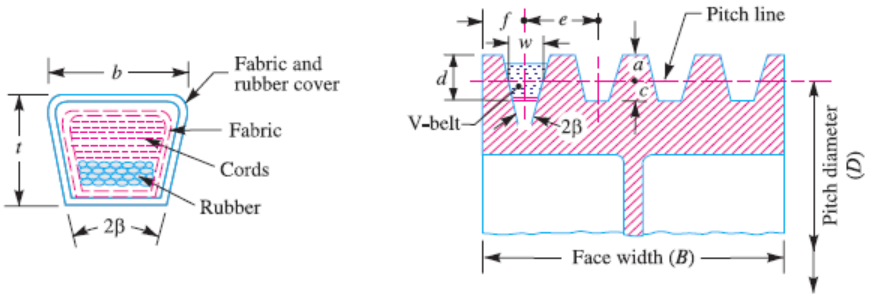
where $(F_1)_a$ = allowable largest tension, lbf
b = belt width, in
F_a = manufacturer's allowed tension, lbf/in
C_p = pulley correction factor (Table 17–4)
C_v = velocity correction factor

The steps in analyzing a flat-belt drive can include (see Ex. 17–1)

1. Find $\exp(f\phi)$ from belt-drive geometry and friction
2. From belt geometry and speed find F_c
3. From $T = 63\,025 H_{nom} K_s n_d / n$ find necessary torque
4. From torque T find the necessary $(F_1)_a - F_2 = 2T/d$
5. From Tables 17–2 and 17–4, and Eq. (17–12) determine $(F_1)_a$.
6. Find F_2 from $(F_1)_a - [(F_1)_a - F_2]$
7. From Eq. (i) find the necessary initial tension F_i
8. Check the friction development, $f' < f$. Use Eq. (17–7) solved for f':

The V-belts derive

The V-belts are made of fabric and cords moulded in rubber and covered with fabric and rubber as shown in Fig. 20.1 (a). These belts are moulded to a trapezoidal shape and are made endless. These are particularly suitable for short drives. The included angle for the V-belt is usually from 30° to 40°. The power is transmitted by the *wedging action between the belt and the V-groove in the pulley or sheave. A clearance must be provided at the bottom of the groove as shown in Fig. 20.1 (b), in order to prevent touching of the bottom as it becomes narrower from wear. The V-belt drive may be inclined at any angle with tight side either at top or bottom. In order to increase the power output, several V-belts may be operated side by side. It may be noted that in multiple V-belt drive, all the belts should stretch at the same rate so that the load is equally divided between them. When one of the set of belts break, the entire set should be replaced at the same time. If only one belt is replaced, the new unworn and unstretched belt will be more tightly stretched and will move with different velocity.



(a) Cross-section of a V-belt. (b) Cross-section of a V-grooved pulley.

Fig. 20.1. V-Belt and V-grooved pulley.

Ratio of Driving Tensions for V-belt

A *V*-belt with a grooved pulley is shown in Fig. 20.2.

Let R_1 = Normal reactions between belts and sides of the groove.

R = Total reaction in the plane of the groove.

μ = Coefficient of friction between the belt and sides of the groove.

Resolving the reactions vertically to the groove, we have

$$R = R_1 \sin\beta + R_1 \sin\beta = 2R_1 \sin\beta$$

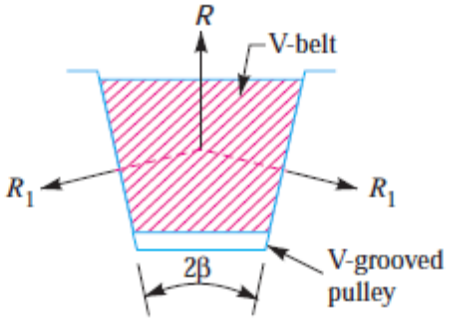


Fig. 20.2. V-belt with pulley.

The ratio of tensions in *V*-belt drive is cosec β times the flat belt drive.

or $$R_1 = \frac{R}{2\sin\beta}$$

We know that the frictional force

$$= 2\,\mu.R_1 = 2\,\mu \times \frac{R}{2\sin\beta} = \frac{\mu.R}{\sin\beta} = \mu.R.\text{cosec}\,\beta$$

Consider a small portion of the belt, as in Art. 18.19, subtending an angle $\delta\theta$ at the centre, the tension on one side will be T and on the other side $(T + \delta T)$. Now proceeding in the same way as in Art. 18.19, we get the frictional resistance equal to μR. cosec β against $\mu.R$. Thus the relation between T_1 and T_2 for the *V*-belt drive will be

$$2.3 \log (T_1 / T_2) = \mu.\theta \text{ cosec } \beta$$

V-flat Drives

In many cases, particularly, when a flat belt is replaced by *V*-belt, it is economical to use flat-faced pulley, instead of large grooved pulley, as shown in Fig. 20.3. The cost of cutting the grooves is thereby eliminated. Such a drive is known as ***V-flat drive.***

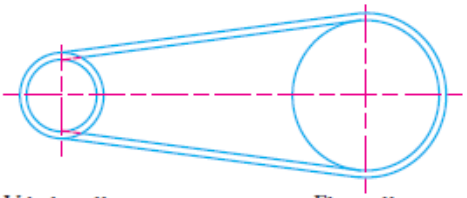


Fig. 20.3. V-flat drive.

CONDITION FOR TRANSMISSION OF MAXIMUM POWER

We have already discussed in Art. 33·13 that the power transmitted by a belt,

$$P = (T_1 - T_2)\, v \qquad ...(i)$$

where T_1 = Tension on the tight side,

T_2 = Tension on the slack side, and

v = Velocity of the belt.

We have also discussed in Art. 33·14 the ratio of tensions,

$$\frac{T_1}{T_2} = e^{\mu\theta} \quad \text{or} \quad T_2 = \frac{T_1}{e^{\mu\theta}} \qquad ...(ii)$$

Substituting the value of T_2 in equation (i),

$$P = \left(T_1 - \frac{T_1}{e^{\mu\theta}}\right) \times v = T_1\left(1 - \frac{1}{e^{\mu\theta}}\right) \times v = T_1 \times v \times C \qquad ...(iii)$$

where $$C = \left(1 - \frac{1}{e^{\mu\theta}}\right)$$

We know that tension in the tight side,

$$T_1 = T - T_C$$

where T = Maximum tension in the belt in newtons, and

T_C = Centrifugal tension in newtons.

Substituting the value of T_1 in equation (iii),

$$P = (T - T_C)\, v\, C = (T - mv^2)\, v\, C = (Tv - mv^2)\, C$$

We know that for maximum power, differentiating the above equation and equating the same to zero,

$$T - 3\, mv^2 = 0 \qquad ...(iv)$$

$$T - 3T_C = 0 \qquad ...(\text{Substituting } mv^2 = T_C)$$

or $$T = 3T_C$$

It shows that when the power transmitted is maximum $\frac{1}{3}$rd of the maximum tension is absorbed as centrifugal tension.

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BELT SPEED FOR MAXIMUM POWER

We have already discussed in Art. 33·17 that for maximum power transmission

$$T - 3\, mv^2 = 0$$

or $$3\, mv^2 = T$$

$$\therefore \quad v = \sqrt{\frac{T}{3m}}$$

where v = Speed of the belt for maximum transmission of power,

T = *Maximum tension in the belt, and

m = Mass of the belt for unit length.

Note: The power transmitted when 1/3 of the maximum tension is absorbed as centrifugal tension (condition of last article) at belt speed for maximum power (condition of the above article) is known as *absolute maximum power or* in other words, maximum power which can be transmitted under any conditions.

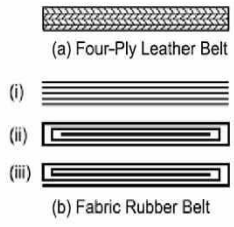
* Maximum tension in the belt is equal to sum of tensions in tight side (T_1) and centrifugal tension (T_C).

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BELT MATERIALS

1. Leather: advantage←high coefficient of friction and consequently high power transmitting capacity
2. Canvas
3. Rubber
4. Rubberised fabric←They are made from several layers of canvas or cotton –duck impregnated with rubber
5. Synthetic materials

BELT CONSTRUCTIONS



Flat Belts

FABRIC RUBBER BELTS

1. The fabric transmits major portion of load.
2. The rubber protects the fabric against damage and increases the coefficient of friction.
3. The presence of rubber allows different plies of fabric to work together as one unit.

ADVANTAGES OF FABRIC RUBBER BELTS

1. They have high load carrying capacity
2. They have long service life.
3. They can operate at high operating speed up to 300m/s

DISADVANTAGES OF FABRIC RUBBER BELTS

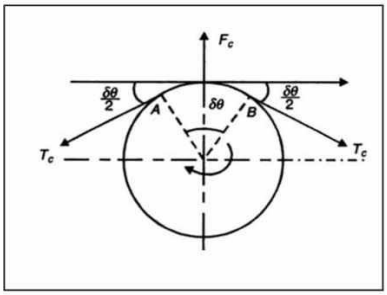
1. They cannot operate on small diameter pulleys.
2. They are subjected to destruction in the presence of mineral oil, gasoline and alkalis.

CENTRIFUGAL TENSION IN BELTS

When a belt operates at high speed ,there is a considerable centrifugal force (inertia force) acting on the belt .This force is developed due to centrifugal action of belt which tries to lift the belt from the pulley. On account of this inertia force, two tensions of equal magnitude are induced on the tight and slack sides of the belt.

Consider a small element of belt which is in equilibrium under the action of the following forces.

1. Centrifugal force F_c
2. Centrifugal tension T_c acting on tight and slack side in addition to normal tension on the tight and slack side.



Let
m = mass of the belt per unit length
r = radius of the pulley
v = peripheral velocity of the belt
and $\delta\theta$ = angle of contact of the element over the pulley

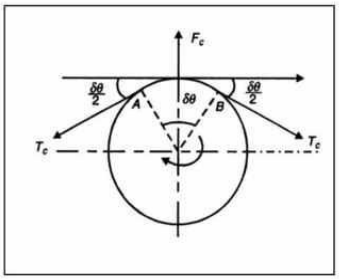
Centrifugal force on the element, F_c

F_c =mass of the element ×acceleration = mr $\delta\theta \times \frac{v^2}{r}$

ie F_c= m$v^2\delta\theta$ ← (1)

For the equilibrium of element AB, let us resolve the force in vertical direction.

$F_c = 2T_C \sin\frac{\delta\theta}{2} \rightarrow F_c = T_C\delta\theta$ (for small angle $\sin\frac{\delta\theta}{2} = \frac{\delta\theta}{2}$)←(2)



From the above two equations (1) and (2)

m$v^2\delta\theta = T_c\ \delta\theta$

Therefore centrifugal tension, T_c =mv^2

Centrifugal tension on the belt is a function of the mass of the belt and peripheral velocity and it is independent of tensions on tight and slack side.

POWER TRANSMISSION CAPACITY

The power transmission capacity of the belt is given by the following relation:

$$\dot{P} = \frac{(T_1 - T_2)v}{1000}\ \text{kW}$$

For the condition of maximum power transmission capacity, T=3mv^2 =3T_c

Therefore for max power to be transmitted, the maximum allowable belt tension should be equal to 3 times the centrifugal tension.

The threshold speed of the belt, $v_{max} = \sqrt{\frac{T}{3m}}$, where **T is the total tension on tight side** $= T_1 + T_C$

The ratio of tensions on tight and slack sides, when the effect of centrifugal tension is considered, $\frac{T_1 - T_C}{T_2 - T_C} = e^{\mu\theta}$

DESIGN PROCEDURE FOR FLAT BELT

STEP 1: Make preliminary decision regarding

a. Material used for belt –generally oak tanned /mineral tanned leather
   Design stress of belt-assume design stress 2.06 MN/m^{2}refer →DB PGNO 291,Eq 14.4
b. Material used for pulley –generally CI,because of damping property.
c. Type of joint required –generally cemented with η_j =100% or Refer table 14.5/page 307
d. Service condition ;service factor ,C_s =1.2 →general or refer table 14.8/page 310
e. Calculate design power ,P_d=P(KW)=PxC$_s$(KW)

STEP 2: diameter of motor pulley (driver)

d= (525-630)$\sqrt[3]{\frac{P(KW)}{\omega_{max}}}$ = (525-630)$\sqrt[3]{\frac{P(KW)}{2\pi n_{max}}}$ where n_{max} =speed in rps (refer DB eqn 14.9(a)/page 292)

DESIGN PROCEDURE OF FLAT BELT DRIVE

STEP 3: diameter of driven pulley considering creep of the belt

D=(1-ϵ)di (DB Eqn 14.9(b)/page 292) ,where i←velocity ratio,where ϵ=0.01to 0.03 (coefficient of creep)

STEP 4: Standardise pulley diameters in steps 2 & 3 using Table 14.13(b)/page 313

Step 5: speed

v=$\frac{\pi D_1 N_1(1-\epsilon)}{60}$,where D_1 ←diameter of driver ,N_1 ←rpm of driver

STEP 6: centre distance

c≥(1.5- 2) (D+d) (refer DB 14.2(d)/page 290)

DESIGN PROCEDURE OF FLAT BELT DRIVE

STEP 7 : Coefficient of friction between pulley &belt using Barths formula

$\mu = 0.54 - \frac{0.712}{2.542+v}$ ←below eqn 14.3(d)/page290

Or using Table 14.2(b)/page 305

If material of belt different select μ from table 14.2(a)/page305

STEP 8: Determine length of belt (open/cross) refer eqn 14.2(b),14.2(c)/page 290

STEP 9: Angle of contact (open/cross) refer Eqn 14.1(a)or 14.1(b)/page 289

DESIGN PROCEDURE OF FLAT BELT DRIVE

Step 10: width of belt (b) ←find standard width from DB table 14.1/page 305

Power transmitted, P (KW)$=\frac{btv}{1000}\left[\sigma_d - \frac{wv^2}{10^6 g}\right]\left(\frac{e^{\mu\theta}-1}{e^{\mu\theta}}\right)\eta_j$ ← *Eqn 14.5(a)/page 291*

η_j = **efficiency** of joint ←table 14.5/page307

$\frac{D_{min}}{t} < 50$; D_{min} refers to smaller diameter

STEP 11: calculate tensions in the belt T_1, T_2 &T_o using

$P_d = P = \frac{(T_1-T_2)v}{1000}$ ←DB Eqn 14.5(a)/page 291

At high velocities; v>10m/sec, we should consider centrifugal tension, otherwise we can neglect it .

$\frac{T_1-T_c}{T_2-T_c} = e^{\mu\theta}$ ← *DB Eqn14.3(c)/page 290*

Relation between initial tension and power tensions; $\sqrt{T_1} + \sqrt{T_2} = 2\sqrt{T_O}$,where T_O is initial tension in N.(refer Eqn 14.8/page 291)

Drums and Ropes

Rope Drives

The rope drives are widely used where a large amount of power is to be transmitted, from one pulley to another, over a considerable distance. It may be noted that the use of flat belts is limited for the transmission of moderate power from one pulley to another when the two pulleys are not more than 8 metres apart. If large amounts of power are to be transmitted, by the flat belt, then it would result in excessive belt cross-section.

The ropes drives use the following two types of ropes :

1. Fibre ropes, and 2. *Wire ropes.

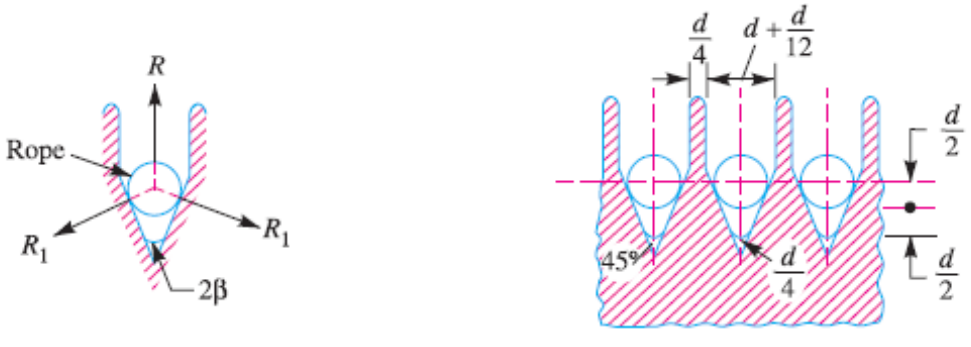
The fibre ropes operate successfully when the pulleys are about 60 metres apart, while the wire ropes are used when the pulleys are upto 150 metres apart.

Notes : 1. The diameter of manila and cotton ropes usually ranges from 38 mm to 50 mm. The size of the rope is usually designated by its circumference or '***girth***'.

2. The ultimate tensile breaking load of the fibre ropes varies greatly. For manila ropes, the average value of the ultimate tensile breaking load may be taken as 500 d^2 kN and for cotton ropes, it may be taken as 350 d^2 kN, where d is the diameter of rope in mm.

Sheave for Fibre Ropes

The fibre ropes are usually circular in cross-section as shown in Fig. 20.6 (*a*). The sheave for the fibre ropes, is shown in Fig. 20.6 (*b*). The groove angle of the pulley for rope drives is usually 45°.



(*a*) Cross-section of a rope. (*b*) Sheave (grooved pulley) for ropes.

Fig. 20.6. Rope and sheave.

Ratio of Driving Tensions for Fibre Rope

A fibre rope with a grooved pulley is shown in Fig. 20.6 (*a*). The fibre ropes are designed in the similar way as *V*-belts. We have discussed in Art. 20.5, that the ratio of driving tensions is

$$2.3 \log \left(\frac{T_1}{T_2} \right) = \mu.\theta \operatorname{cosec} \beta$$

where μ, θ and β have usual meanings.

RATIO OF TENSIONS IN ROPE DRIVE

We have already discussed in Art. 33·14 the ratio of tensions in a belt drive. The same principle may be used for the determination of ratio of tensions in case of a rope drive. Now consider a rope running in a groove as shown in Fig. 33·14.

Let R_1 = Normal reactions between rope and sides of the groove.

R = Total reaction in the plane of the groove.

2α = Angle of the groove.

μ = Coefficient of friction between rope and sides of the groove.

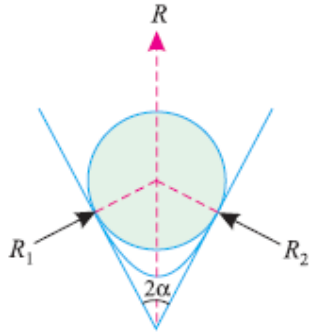


Fig. 33.14. Ratio of tensions in rope drive.

Resolving the reactions vertically to the groove.

$$R = R_1 \sin \alpha + R_1 \sin \alpha = 2R_1 \sin \alpha$$

or $$R_1 = \frac{R}{2 \sin \alpha}$$

We know that the frictional force

$$= 2\mu R_1 = 2\mu \times \frac{R}{2 \sin \alpha} = \frac{\mu R}{\sin \alpha} = \mu R \operatorname{cosec} \alpha$$

Now consider a small portion of the belt as in Art. 33·14 subtending an angle $\delta\theta$ at the centre. The tension on one side will be T and on the other side $T + \delta T$. Now proceeding as in Art. 33·14, we get the frictional resistance equal to (μR cosec α) against μR. Thus the relation between T_1 and T_2 for the rope drive will be:

$$2.3 \log \left(\frac{T_1}{T_2} \right) = \mu\theta \operatorname{cosec} \alpha.$$

Stresses in Wire Ropes

A wire rope is subjected to the following types of stresses :

1. ***Direct stress due to axial load lifted and weight of the rope***

Let W = Load lifted,
w = Weight of the rope, and
A = Net cross-sectional area of the rope.

$\therefore$ Direct stress, $\sigma_d = \dfrac{W + w}{A}$

2. ***Bending stress when the rope winds round the sheave or drum.*** When a wire rope is wound over the sheave, then the bending stresses are induced in the wire which is tensile at the top and compressive at the lower side of the wire. The bending stress induced depends upon many factors such as construction of rope, size of wire, type of centre and the amount of restraint in the grooves. The approximate value of the bending stress in the wire as proposed by Reuleaux, is

$$\sigma_b = \frac{E_r \times d_w}{D}$$

and equivalent bending load on the rope,

$$W_b = \sigma_b \times A = \frac{E_r \times d_w \times A}{D}$$

where E_r = Modulus of elasticity of the wire rope,
d_w = Diameter of the wire,
D = Diameter of the sheave or drum, and
A = Net cross-sectional area of the rope.

It may be noted that E_r is not the modulus of elasticity for the wire material, but it is of the entire rope. The value of E_r may be taken as 77 kN/mm^2 for wrought iron ropes and 84 kN/mm^2 for steel ropes. It has been found experimentally that $E_r = 3/8\ E$, where E is the modulus of elasticity of the wire material.

If σ_b is the bending stress in each wire, then the load on the whole rope due to bending may be obtained from the following relation, *i.e.*

$$W_b = \frac{\pi}{4} (d_w)^2\, n \times \sigma_b$$

where n is the total number of wires in the rope section.

3. ***Stresses during starting and stopping.*** During starting and stopping, the rope and the supported load are to be accelerated. This induces additional load in the rope which is given by

$$W_a = \frac{W + w}{g} \times a \qquad \text{...(W and w are in newton)}$$

and the corresponding stress,

$$\sigma_a = \frac{W + w}{g} \times \frac{a}{A}$$

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where a = Acceleration of the rope and load, and

g = Acceleration due to gravity.

If the time (t) necessary to attain a speed (v) is known, then the value of 'a' is given by

$$a = v / 60\ t$$

The general case of starting is when the rope has a slack (h) which must be overcome before the rope is taut and starts to exert a pull on the load. This induces an impact load on the rope.

The impact load on starting may be obtained by the impact equation, *i.e.*

$$W_{st} = (W + w)\left[1 + \sqrt{1 + \frac{2a \times h \times E_r}{\sigma_d \times l \times g}}\right]$$

and velocity of the rope (v_r) at the instant when the rope is taut,

$$v_r = \sqrt{2a \times h}$$

where a = Acceleration of the rope and load,

h = Slackness in the rope, and

l = Length of the rope.

When there is no slackness in the rope, then $h = 0$ and $v_r = 0$, therefore

Impact load during starting,

$$W_{st} = 2\ (W + w)$$

and the corresponding stress,

$$\sigma_{st} = \frac{2\ (W + w)}{A}$$

4. ***Stress due to change in speed.*** The additional stress due to change in speed may be obtained in the similar way as discussed above in which the acceleration is given by

$$a = (v_2 - v_1) / t$$

where $(v_2 - v_1)$ is the change in speed in m/s and t is the time in seconds.

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5. ***Effective stress.*** The sum of the direct stress (σ_d) and the bending stress (σ_b) is called the effective stress in the rope during normal working. Mathematically,

Effective stress in the rope during normal working

$$= \sigma_d + \sigma_b$$

Effective stress in the rope during starting

$$= \sigma_{st} + \sigma_b$$

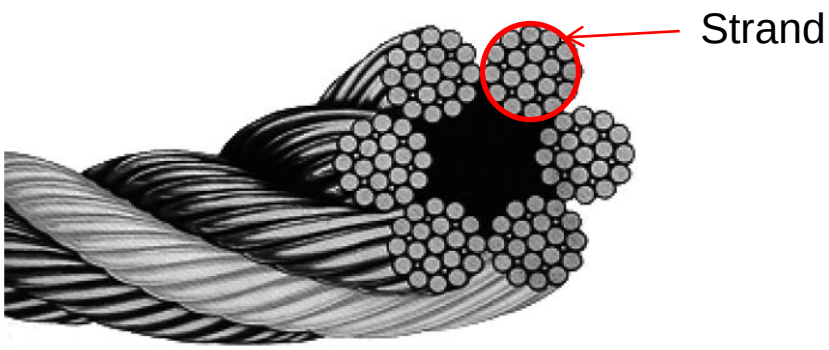
and effective stress in the rope during acceleration of the load

$$= \sigma_d + \sigma_b + \sigma_a$$

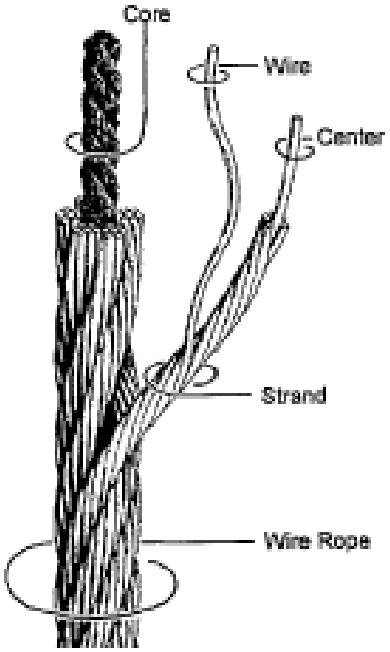
2.2.Drums and Ropes

Wire Ropes

Wire rope consists of multiple wires or strands, helically wound or laid about an axis.
Wires, strands and cores are independent- that interact with each other during service.



Wire ropes are designed in different steel grades, finishes and a variety of constructions to attain the best balance of strength, abrasion resistance, crush resistance, bending fatigue resistance and corrosion resistance for each application.



Factors Influencing the lifetime of wire ropes:

- type of wire ropes;
- diameter of the drums and sheaves;
- reverse bending
- speeds with which the wire ropes run through the sheaves
- ratio between normal working load and the maximum load in the wire ropes;
- safety factor; being the ratio between the breaking load and the normal working load
- choice of the hardness in the groove of the sheaves;
- fleet-angle between the wire rope and the sheave, respectively between the drum groove and wire rope;
- greasing or lubrication of the wire ropes and the frequency of greasing or lubrication;
- mechanical damage

<u>Selection of drum and sheave diameter</u>

For faster operation of ropes in cranes:

$$D/d=30$$

For fast un-loaders:

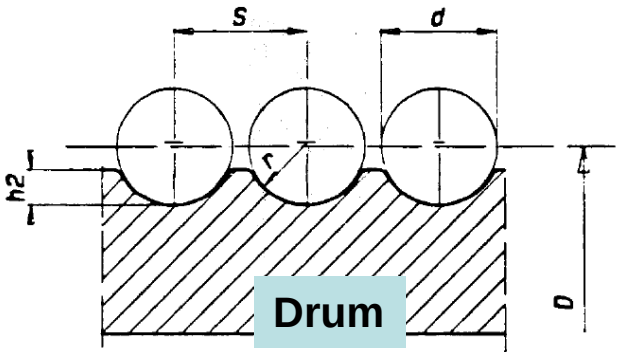
$$D/d= 36$$

Where,

D= sheave diameter or drum diameter, centre to centre of wire rope,

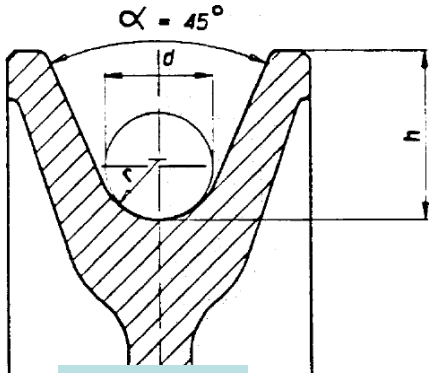
d= wire rope diameter.

Dimension of grooves on drums and in sheaves:



Drum

$$h_2 \approx 0{,}3d$$
$$r \approx 1{,}05 \cdot (d:2)$$
$$S_{\min} \approx 1{,}1d$$

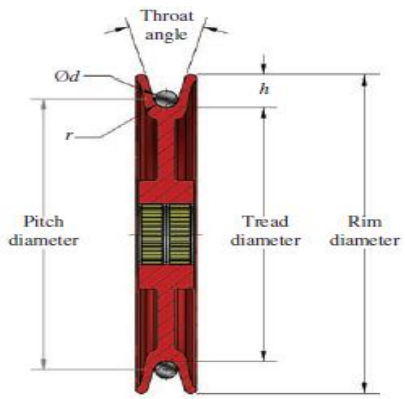


Sheave

$$h = 1{,}5d \text{ to } 2d$$
$$r \approx 1{,}05(d:2)$$

Sheave and Drum Design





(a) Sheave with roller bearing (b) Sheave dimensional specifications

Design of a roller bearing wire rope sheave

A suggested and minimum diameter (D) of the sheave or drum is selected based on the wire rope diameter (d) and the construction of the wire rope. This diameter is called the tread diameter (D) and is where the wire rope rides on the sheave or grooved drum. The D/d ratio measures how tightly the wire rope bends over a sheave. The smaller the ratio, the tighter the bend in the wire rope.
The suggested and minimum D/d ratios based on wire rope construction are shown in Table 7–22. Figure 7–48The dimensional specifications and terminology of the sheave. The tread diameter (*D*) is based on the suggested minimum *D/d* ratio. The sheave throat angle should be between 35° and 45°. The groove depth of a sheave should be between

$h_{min} = 1.50 * d$ and $h_{max} = 1.75 * d$

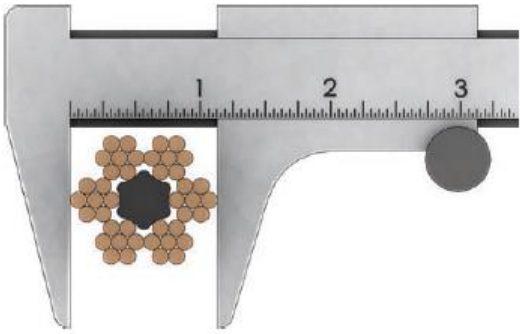


FIGURE 7–42 Correct method of measurement of a wire rope

TABLE 7–20 Wire Rope Classification

Classification	Wires per strand	Maximum number of outer wires in strand
6×7	7 through 15	9
6×19	16 through 26	12
6×36	27 through 49	18
6×61	50 through 74	24

*Classifications are the same in 7 and 8 strand wire ropes

TABLE 7–22 Sheave and Drum Diameter Factors

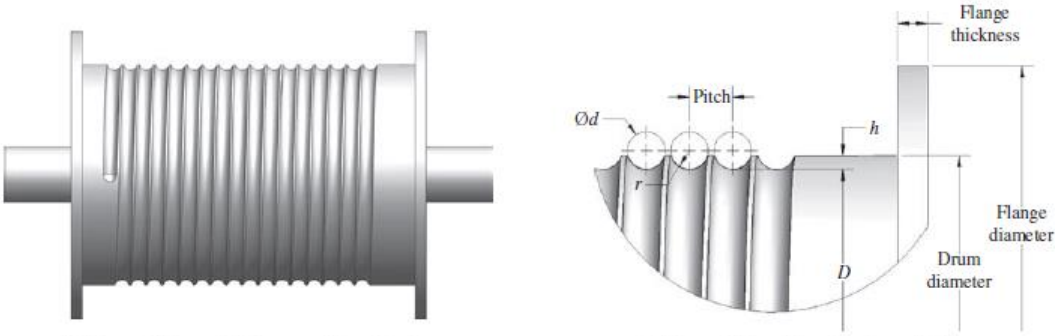
Construction	Suggested *D/d* ratio	Minimum *D/d* ratio
6×7	72	42
6×19 Seale	51	34
6×21 Filler Wire	45	30
6×25 Filler Wire	39	26
6×31 Warrington Seale	39	26
6×36 Warrington Seale	35	23
6×41 Seale Filler Wire	20	20
6×41 Warrington Seale	32	21
6×42 Tiller	21	14
8×19 Seale	41	27
8×25 Filler Wire	32	21

the next adjacent groove along the pitch diameter of the wire rope. The pitch distance should range between:

$$\text{Pitch} = p = 2.065 * r \text{ and } p = 2.18 * r$$

The minimum groove depth (h) is the distance from the drum diameter to the tread diameter (D) on a helically grooved drum. This distance is given by

$$h = 0.374 * d$$



(*a*) Grooved drum with flanges and journal ends

(*b*) Grooved drum dimensional specifications

Design details for a grooved wire rope drum

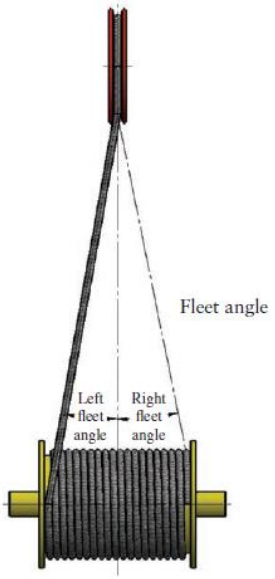


TABLE 7–24 Grades of Wire Rope

Grade	Tensile strength
Plow Steel	1570 N/mm^2
Improved Plow Steel	1770 N/mm^2
Extra Improved Plow Steel	1960 N/mm^2

Design Factors and Working Loads:

Industry standards and regulations require minimum design factors to be applied when using wire rope. The design factor is applied to the minimum breaking force of the wire rope.
A minimum design factor (SF) of 5 is used in overhead cranes, gantry cranes, and overhead
hoists. The maximum working load of a wire rope can then be determined in the following equation.

Maximum working load = Minimum breaking force/SF

This allowable working load is the load the wire rope is expected to carry not only due to static loading, but also loading resulting from accelerations and shock loading conditions on a system. Shock loads can be significantly higher than static loads. Shock loading can occur when there is a sudden change or jerking movement of the load and should be avoided.
The technical data are the same for both 6*19 and 6*36 classification of wire rope and are shown in Table 7–25.

TABLE 7–25 6x19 and 6x36 Classes Technical Data

	Fiber core			IWRC		
Diameter	Weight per foot	Min breaking force		Weight per foot	Min breaking force	
		IPS	XIP		IPS	XIP
in	lb/ft	tons	tons	lb/ft	tons	tons
1/4	0.105	2.74	3.02	0.116	2.94	3.4
5/16	0.164	4.26	4.69	0.18	4.58	5.27
3/8	0.236	6.1	6.72	0.26	6.56	7.55
7/16	0.32	8.27	9.1	0.35	8.89	10.2
1/2	0.42	10.7	11.8	0.46	11.5	13.3
9/16	0.53	13.5	14.9	0.59	14.5	16.8
5/8	0.66	16.7	18.3	0.72	17.9	20.6
3/4	0.95	23.8	26.2	1.04	25.6	29.4
7/8	1.29	32.2	35.4	1.42	34.6	39.8
1	1.68	41.8	46	1.85	44.9	51.7
1 1/8	2.13	52.6	57.8	2.34	56.5	65
1 1/4	2.63	64.6	71.1	2.89	69.4	79.9
1 3/8	3.18	77.7	85.5	3.5	83.5	96
1 1/2	3.78	92	101	4.16	98.9	114
1 5/8	4.44	107	118	4.88	115	132
1 3/4	5.15	124	137	5.67	133	153
1 7/8	5.91	141	156	6.5	152	174
2	6.72	160	176	7.39	172	198
2 1/8	7.59	179	197	8.35	192	221
2 1/4	8.51	200	220	9.36	215	247

Sprocket and chains

Design of Chain Drives

The rating of chain for its power transmission capacity considers three modes of failure:

(1) fatigue of the link plates due to the repeated application of the tension in the tight side of the chain,

(2) impact of the rollers as they engage the sprocket teeth, and

(3) galling between the pins of each link and the bushings on the pins.

Manufacturers recommend the type of lubrication method for given combinations of chain size, sprocket size, and speed. Details are discussed later.

Tables 7–14 to 7–16 list the rated power for three sizes of standard chain: no. 40 (1/2 in), no. 60 (3/4 in), and no. 80 (1.00 in).

DESIGN GUIDELINES FOR CHAIN DRIVES ▼

The following are general recommendations for designing chain drives:

1. The minimum number of teeth in a sprocket should be 17 unless the drive is operating at a very low speed, under 100 rpm.

2. The maximum speed ratio should be 7.0, although higher ratios are feasible. Two or more stages of reduction can be used to achieve higher ratios.

3. The center distance between the sprocket axes should be approximately 30 to 50 pitches (30–50 times the pitch of the chain).

4. The larger sprocket should normally have no more than 120 teeth.

5. The preferred arrangement for a chain drive is with the centerline of the sprockets horizontal and with the tight side on top.

6. The chain length must be an integral multiple of the pitch, and an even number of pitches is recommended.

The center distance should be made adjustable to accommodate the chain length and to take up for tolerances and wear. Excessive sag on the slack side should be avoided, especially on drives that are not horizontal.

A convenient relation between center distance (CD), chain length (Lc), number of teeth in the small sprocket (N1), and number of teeth in the large sprocket (N2), expressed in pitches, is:

- **Chain Length in Pitches**

$$L_c = 2CD + \frac{N_2 + N_1}{2} + \frac{(N_2 - N_1)^2}{4\pi^2 CD} \qquad (7\text{–}18)$$

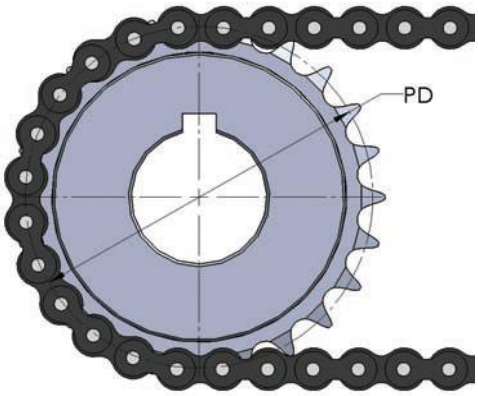
The center distance for a given chain length, again *in pitches*, is

- **Center Distance in Pitches**

$$CD = \frac{1}{4}\left[L_c - \frac{N_2 + N_1}{2} + \sqrt{\left[L_c - \frac{N_2 + N_1}{2}\right]^2 - \frac{8(N_2 - N_1)^2}{4\pi^2}}\right] \qquad (7\text{–}19)$$

The computed center distance assumes no sag in either the tight or the slack side of the chain, and thus it is a maximum. Negative tolerances or adjustment must be provided. Adjustment for wear must also be provided.

7. The pitch diameter of a sprocket with N teeth for a chain with a pitch of p is



Pitch Diameter of Sprocket

$$PD = \frac{p}{\sin(180°/N)} \text{ inches or mm} \qquad (7\text{–}20)$$

8. The minimum sprocket diameter and therefore the minimum number of teeth in a sprocket are often limited by the size of the shaft on which it is mounted. Check the sprocket catalog.
9. The arc of contact, θ_1, often called the *angle of wrap*, of the chain on the smaller sprocket should be greater than 120°.

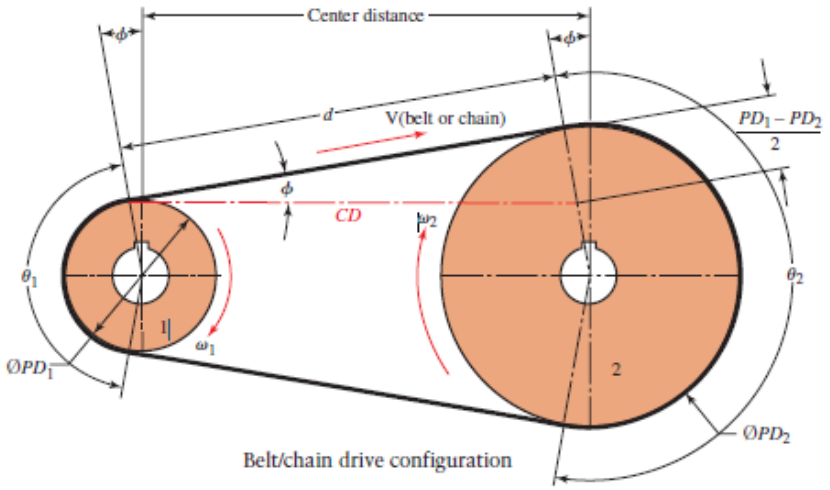
Angle of Wrap Smaller Sprocket

$$\theta_1 = 180° - 2\sin^{-1}[(PD_2 - PD_1)/2CD] \qquad (7\text{–}21)$$

10. For reference, the arc of contact, θ_2, on the larger sprocket is

Angle of Wrap Large Sprocket

$$\theta_2 = 180° + 2\sin^{-1}[(PD_2 - PD_1)/2CD] \qquad (7\text{–}22)$$



Belt/chain drive configuration

the basic layout where ω1 is equal to the input angular velocity (ω driving) and v2 is equal to the output angular velocity (ω driven).

$$v_{\frac{belt}{chain}} = \frac{PD_{driving}}{2} \cdot \omega_{driving} = \frac{PD_{driven}}{2} \cdot \omega_{driven} \quad (7\text{–}2)$$

Letting the driving member to be 1 as used in Figure 7–4 and the driven member to be 2, we can restate Equation (7–2) as

$$v_{\frac{belt}{chain}} = \frac{PD_1}{2} \cdot \omega_1 = \frac{PD_2}{2} \cdot \omega_2 \quad (7\text{–}3)$$

also be expressed by the ratios of the pitch diameters or the numbers of teeth of the driving and driven sprockets.

$$VR = \frac{\omega_{driving}}{\omega_{driven}} = \frac{PD_{driven}}{PD_{driving}} = \frac{N_{driven}}{N_{driving}} \quad (7\text{–}4)$$

The angle of wrap on the driving and driven sprockets can be found from,

$$\sin(\varnothing) = \frac{\left(\frac{PD_2 - PD_1}{2}\right)}{CD} \quad (7\text{–}5)$$

The angle of wrap on sprocket (1)

$$\theta_1 = 180^\circ - 2 \cdot \varnothing \quad (7\text{–}6)$$

The angle of wrap on sprocket (2)

$$\theta_2 = 180^\circ + 2 \cdot \varnothing \quad (7\text{–}7)$$

The length of the belt or chain wrap on sprocket 1 and sprocket 2 is the arc length s_1 and s_2.

$$s_1 = \frac{PD_1}{2} \cdot \theta_1 \qquad (7\text{–}8)$$

$$s_2 = \frac{PD_2}{2} \cdot \theta_2 \qquad (7\text{–}9)$$

where θ_1 and θ_2 are in radians.
The distance, d, represents the belt or chain length that is tangent to sprocket 1 and sprocket 2.

$$d = CD \cdot \cos\varnothing \qquad (7\text{–}10)$$

The belt or chain perimeter length, L_p, can now be calculated.

Belt or chain perimeter length $= L_p$

$$L_p = 2 \cdot d + s_1 + s_2 \qquad (7\text{–}11)$$

The distance, d, is often called the *span* of the belt or chain drive system, giving the unsupported length of the belt or chain. Long spans sometimes lead to vibration during operation, called *whip*, and should be avoided where possible.

Brakes

They are machine elements that absorb either kinetic or potential energy in the process of slowing down or stopping a moving part.

Brake capacity depends upon :

1. the unit pressure between the braking surfaces.
2. The coefficient of friction.
3. The ability of the brake to dissipate heat equivalent to energy being absorbed.

A brake is a device used to bring a moving system to rest, to slow its speed, or to control its speed to a certain value under varying conditions.
A clutch is a device used to connect or disconnect a driven component from the prime mover of the system.

• A clutch is a device used to connect or disconnect a driven component from the prime mover of the system A brake is a device used to bring a moving system to rest, to slow its speed, or to control its speed to a certain value under varying conditions.

Clutches and brakes that use friction surfaces as the means of transmitting the torque to start or stop a mechanism can be classified according to the general geometry of the friction surfaces and by the means used to actuate them. In some cases, the same basic geometry can be used as either a clutch or a brake by selectively attaching the friction elements to the driver, the driven machine, or the stationary frame of the machine.
The following types of clutches and brakes are sketched in Figure 22-2:

The mechanical brakes are primarily of three types:

- Drum brake .

  Internal expanding shoes brake.

  External contracting shoes brake.

- Disc brake.
- Band brake .

FIGURE 22–2 Types of friction clutches and brakes [(b), Tol-O-Matic, Hamel, MN]

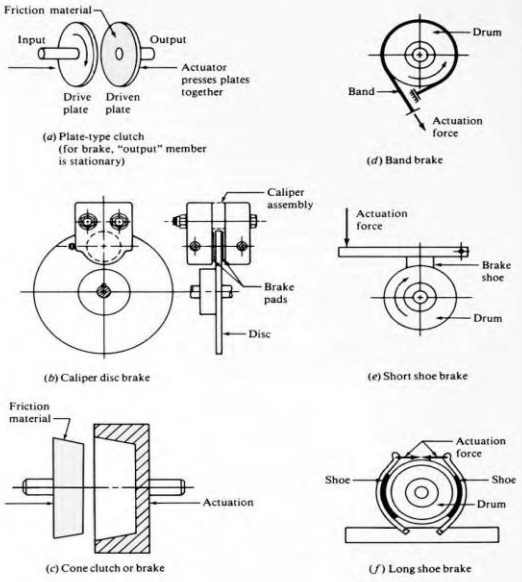


FIGURE 22–3 Manually operated clutch (Rockford Division, Borg-Warner Corp., Rockford, IL)

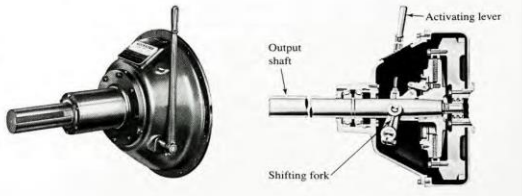


FIGURE 22–4 Spring-applied, electrically released, long shoe brake (Eaton Corp., Cutler-Hammer Products, Milwaukee, WI)

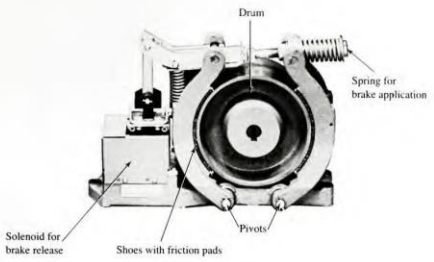
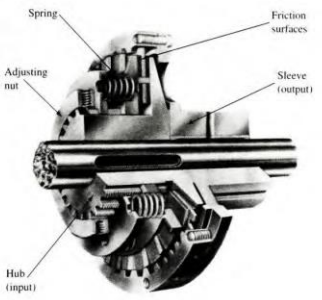


FIGURE 22–5 Slip clutch. Springs apply normal pressure on the friction plates. The spring force is adjustable to vary the torque level at which clutch will slip. (The Hilliard Corp., Elmira, NY)



Spring-applied. Sometimes called a *fail-safe* design when applied to a brake, the brake is applied automatically by springs unless there is some opposing force present. Thus, if power fails, or if air pressure or hydraulic pressure is lost, or if the operator is unable to perform the function, the springs apply the brake and stop the load. The concept may also be used to engage or to disengage a clutch.

Centrifugal. A centrifugal clutch is sometimes employed to permit the driving system to accelerate without a connected load. Then, at a preselected speed, centrifugal force moves the clutch elements into contact to connect the load. As the system slows, the load will be automatically disconnected.

The parameters involved in the rating of clutches and brakes are described in the following

1. Torque required to accelerate or decelerate the system
2. Time required to accomplish the speed change
3. The cycling rate: number of on/off cycles per unit time
4. The inertia of the rotating or translating parts
5. The environment of the system: temperature, cooling effects, and so on
6. Energy-dissipation capability of the clutch or the brake
7. Physical size and configuration
8. Actuation means
9. Life and reliability of the system
10. Cost and availability

Two basic methods are used to determine the torque capacity required of a clutch or a brake. One relates the capacity to the power ofthe motor driving the system. Recall that,

in general, power = torque X rotational speed {P = Tn). The required torque capacity is then usually expressed in the form

$$\text{Required Torque Capacity of Clutch or Brake} = T = \frac{CPK}{n} \qquad (22\text{–}1)$$

where C = conversion factor for units
K = service factor based on the application

The value for the K factor in the torque equation is largely a design decision. Some ofthe typical guidelines follow:

1. For brakes under average condifions, use ^ = 1.0.
2. For clutches in light duty where the output shaft does not assume its normal load until after it is up to speed, use K = 1.5.
3. For clutches in heavy duty where large attached loads must be accelerated, use K - 3.0.

CALIPER DISC BRAKES

The disc brake pads are brought into contact with the rotating disc by fluid pressure acting on a piston in the caliper The pads are either round or in a short crescent shape to cover more of the disc surface [see Figure 22-2(b) and Figure 22-7]. Designs for the friction torque and for the wear rating are similar to those explained for plate-type brakes.

CONE CLUTCH OR BRAKE DESIGN

The angle of inclination of the conical surface of the cone clutch or brake is typically 12°.

Referring to Figure 22-16, we see that as an axial force F,, is applied by a spring, manually or by fluid pressure, a normal force N is created between the mating friction surfaces, all around the periphery of the cone. The desired friction force Ff 'is produced in the tangential direction, where Ff = f N. It is assumed that the friction force acts at the mean radius of the co

$$T_f = F_f R_m = fNR_m \qquad (22\text{–}8)$$

$$F_f' = fN$$

In addition to the tangentially directed friction force, a friction force develops along the surface of the cone and opposes the tendency for the member having the internal cone surface to move axially away from the external cone. We will call this force F'j and compute it also from

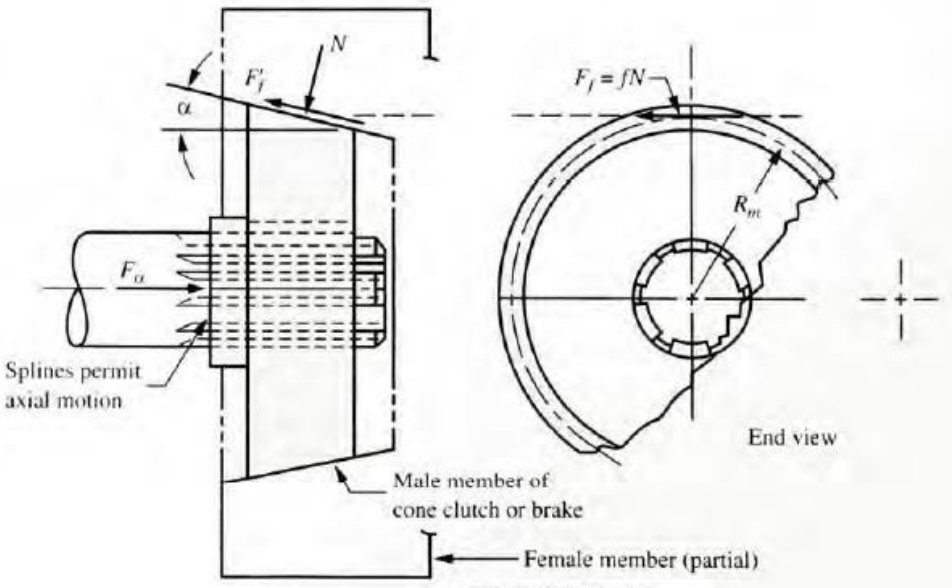


FIGURE 22–16
Cone clutch or brake

For the equilibrium condition of the external cone, the sum of the horizontal forces must be zero. Then

$$F_a = N \sin \alpha + F'_f \cos \alpha = N \sin \alpha + fN \cos \alpha = N(\sin \alpha + f \cos \alpha)$$

or

$$N = \frac{F_a}{\sin \alpha + f \cos \alpha} \tag{22–12}$$

Substituting this into Equation (22–8) gives

Frictional Torque on a Cone Clutch or Brake

$$T_f = \frac{f R_m F_a}{\sin \alpha + f \cos \alpha} \tag{22–13}$$

Short Shoe Drum Brakes

Figure 22-17 shows a sketch of a drum brake in which the actuating force W acts on the lever that pivots on pin A. This creates a normal force between the shoe and the rotating drum

The objectives of the analysis are to determine the relationship between the applied load and the friction force and to be able to evaluate the effect of design decisions such as the size of the drum, the lever dimensions, and the placement of the pivot A.

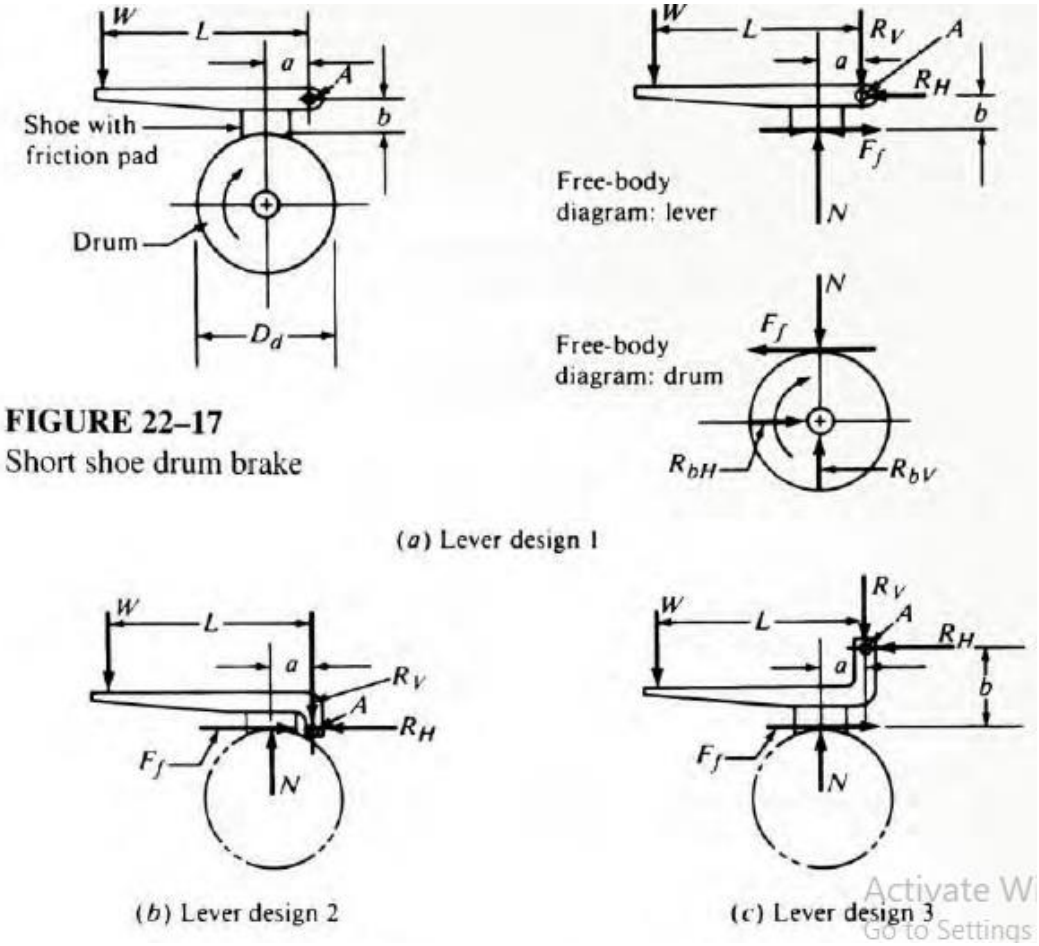


FIGURE 22–17
Short shoe drum brake

Activate W
Go to Settings

diagrams in Figure 22–17(a) support this analysis. For the lever, we can sum moments about the pivot A:

$$\Sigma M_a = 0 = WL - Na + F_f b \tag{22–14}$$

But note that $F_f = fN$ or $N = F_f/f$. Then

$$0 = WL - F_f a/f + F_f b = WL - F_f(a/f - b)$$

Solving for W gives

$$W = \frac{F_f(a/f - b)}{L} \tag{22–15}$$

Solving for F_f gives

Friction Force on Drum Brake

$$F_f = \frac{WL}{(a/f - b)} \tag{22–16}$$

We can use these equations for the friction torque by noting

Friction Torque

$$T_f = F_f D_d/2 \tag{22–17}$$

where D_d = diameter of the drum

Note the alternate positions of the pivot in Parts (b) and (c) of Figure 22–17. In (b), the dimension $b = 0$.

Long Shoe Drum Brakes

The assumption used for short shoe brakes, that the resultant friction force acted at the middle of the shoe, cannot be used in the case of shoes covering more than about 45° of the drum.

In such cases, the pressure between the friction lining and the drum is very nonuniform, as is the moment of the friction force and of the normal force with respect to the pivot of the shoe.

The following equations govern the performance of a long shoe brake, using the terminology from Figure 22-19.

1. *Friction torque on drum:*

$$T_f = r^2 fw p_{max}(\cos\theta_1 - \cos\theta_2) \tag{22–18}$$

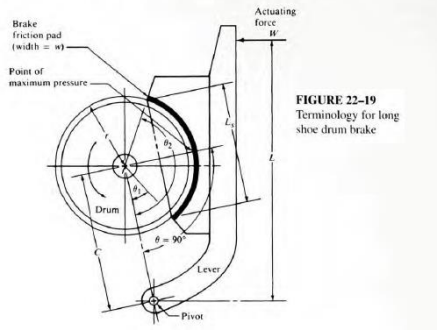


FIGURE 22–19
Terminology for long shoe drum brake

2. ***Actuation force:***

$$W = (M_N + M_f)/L \qquad \textbf{(22–19)}$$

where M_N = moment of normal force with respect to the hinge pin

$$M_N = 0.25 p_{max} wrC[2(\theta_2 - \theta_1) - \sin 2\theta_2 + \sin 2\theta_1] \qquad \textbf{(22–20)}$$

M_f = moment of friction force with respect to the hinge pin

$$M_f = f p_{max} wr[r(\cos\theta_1 - \cos\theta_2) + 0.25C(\cos 2\theta_2 - \cos 2\theta_1)] \qquad \textbf{(22–21)}$$

The sign of M_f is negative (−) if the drum surface is moving away from the pivot and positive (+) if it is moving toward the pivot.

3. ***Friction power:***

$$P_f = T_f n/63\,000 \text{ hp} \qquad \textbf{(22–22)}$$

where n = rotational speed in rpm

4. ***Brake shoe area*** (*Note*: Projected area is used):

$$A = L_s w = 2wr \sin[(\theta_2 - \theta_1)/2] \qquad \textbf{(22–23)}$$

5. ***Wear ratio:***

$$WR = P_f/A \qquad \textbf{(22–24)}$$

BAND BRAKES :

Figure 22-20 shows the typical configuration of a band brake. The flexible band, usually made of steel, is faced with a friction material that can conform to the curvature of the drum.

The application of a force to the lever puts tension in the band and forces the friction materlal against the drum. The normal force, thus created, causes the friction force tangential to the drum surf'ace to be created, retarding the drum. The tension in the band decreases from the value P1 at the pivot side of the band to P2 at the lever side. The net torque on the drum is then

$$T_f = (P_1 - P_2)r \qquad \textbf{(22–25)}$$

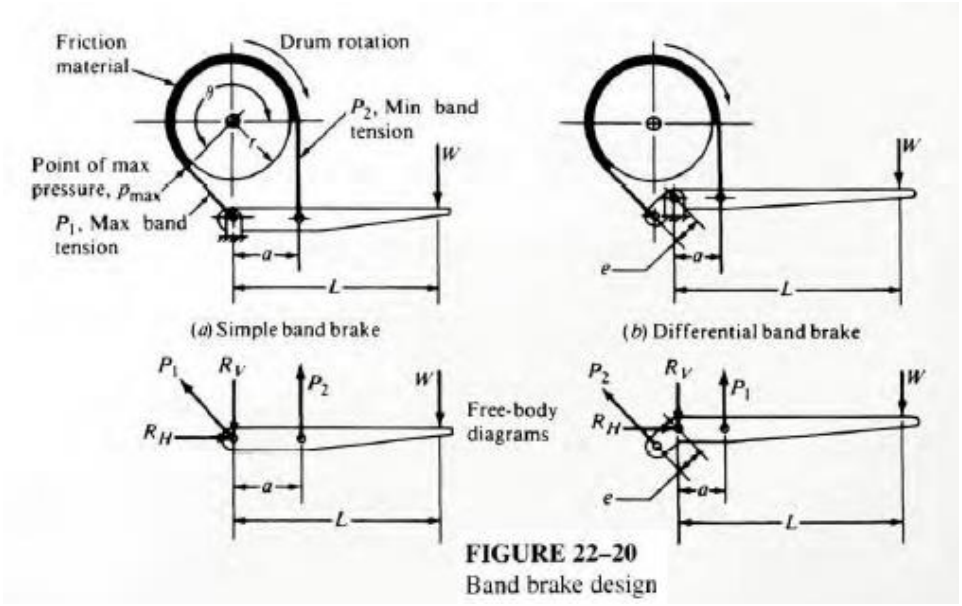


FIGURE 22–20
Band brake design

where r = radius of the drum

The relationship between P_1 and P_2 can be shown (see Reference 4) to be the logarithmic function

$$P_2 = P_1/e^{f\theta} \tag{22–26}$$

where θ = total angle of coverage of the band in radians

The point of maximum pressure on the friction material occurs at the end nearest the highest tension, P_1, where

$$P_1 = p_{max} r w \tag{22–27}$$

and w is the width of the band.

For the two types of band brakes shown in Figure 22–20, the free-body diagrams of the levers can be used to show the following relationships for the actuating force, W, as a function of the tensions in the band. For the simple band brake of Figure 22–20(a),

$$W = P_2 a/L \tag{22–28}$$

The style shown in Figure 22–20(b) is called a *differential band brake*, where the actuation force is

$$W = (P_2 a - P_1 e)/L \tag{22–29}$$

3.2.Clutches

Friction Clutches

A friction clutch has its principal application in the transmission of power of shafts and machines which must be started and stopped frequently. The force of friction is used to start the driven shaft from rest and gradually brings it up to the proper speed without excessive slipping of the friction surfaces. In automobiles, friction clutch is used to connect the engine to the driven shaft. In operating such a clutch, care should be taken so that the friction surfaces engage easily and gradually brings the driven shaft up to proper speed. The proper alignment of the bearing must be maintained and it should be located as close to the clutch as possible. It may be noted that.

1. The contact surfaces should develop a frictional force that may pick up and hold the load with reasonably low pressure between the contact surfaces.
2. The heat of friction should be rapidly dissipated and tendency to grab should be at a minimum.
3. The surfaces should be backed by a material stiff enough to ensure a reasonably uniform distribution of pressure.

The friction clutches of the following types are important from the subject point of view :

i. Disc or plate clutches (single disc or multiple disc clutch),

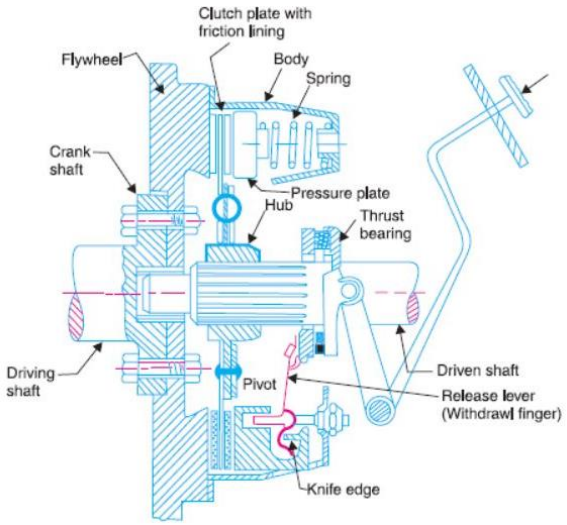
ii. Cone clutches, and

iii. Centrifugal clutches.

Single Disc or Plate Clutch

A single disc or plate clutch, as shown in **Figure(1)**, consists of a clutch plate whose both sides are faced with a friction material (usually of Ferrodo). It is mounted on the hub which is free to move axially along the splines of the driven shaft. The pressure plate is mounted inside the clutch body which is bolted to the flywheel. Both the pressure plate and the flywheel rotate with the engine crankshaft or the driving shaft. The pressure plate pushes the clutch plate towards the flywheel by a set of strong springs which are arranged radially inside the body. The three levers (also known as release levers or fingers) are carried on pivots suspended from the case of the body.

The axial pressure exerted by the spring provides a frictional force in the circumferential direction when the relative motion between the driving and driven members tends to take place . If the torque due to this frictional force exceeds the torque to be transmitted, then no slipping takes place and the power is transmitted from the driving shaft to the driven shaft.



Figure(1)

Now consider two friction surfaces, maintained in contact by an axial thrust *W*, as shown in Figure (2).

T = Torque transmitted by the clutch,

p = Intensity of axial pressure with which the contact surfaces are held together,

r_1 and r_2 = External and internal radii of friction faces, and

μ = Coefficient of friction.

Consider an elementary ring of radius r and thickness d_r as shown in Figure (2)

We know that area of contact surface or friction surface, = 2 π r.dr

∴ Normal or axial force on the ring, or axial force on the ring,

δW = Pressure × Area = p × 2 π r.dr

and the frictional force on the ring acting tangentially at radius r,

Fr = μ.δW = μ.p × 2 π r.dr

∴ Frictional torque acting on the ring, Tr = Fr × r = μ.p × 2 π r.dr × r = 2 π × μ .p.r^2 dr

We shall now consider the following two cases :

1. **When there is a uniform pressure, and**
2. **When there is a uniform wear.**

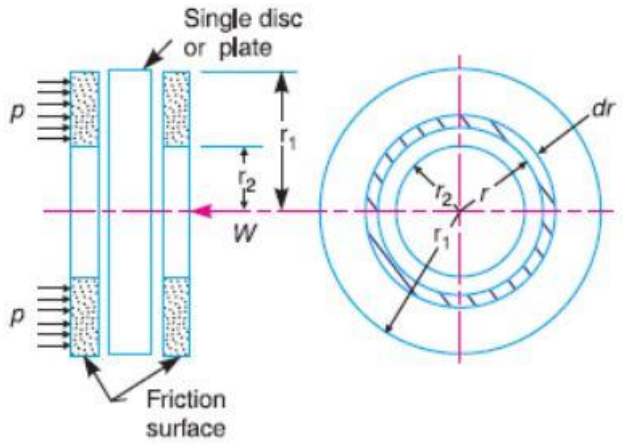


Figure (2)

Considering uniform pressure

When the pressure is uniformly distributed over the entire area of the friction face, then the intensity of pressure,

$$P = \frac{W}{\pi(r_1^2 - r_2^2)} \quad \text{.....(1)}$$

where : **W** = Axial thrust with which the contact or friction surfaces are held together.

We have discussed above that the **frictional torque** on the elementary ring of radius **r** and thickness **dr** is

$$\mathbf{Tr = 2\,\pi\,\mu.p.r^2\,dr}$$

Integrating this equation within the limits from r_2 to r_1 for the total frictional torque .

∴ Total frictional torque acting on the friction surface or on the clutch,

$$T = \int_{r_2}^{r_1} 2\pi\mu.P.r^2.dr = 2\pi\mu.P\left[\frac{r^3}{3}\right]_{r_2}^{r_1} = 2\pi\mu.P\left[\frac{r_1^3 - r_2^3}{3}\right]$$

Substituting the value of **p** from equation (1),

$$T = 2\pi\mu \times \frac{W}{\pi(r_1^2 - r_2^2)} \times \left[\frac{r_1^3 - r_2^3}{3}\right] \quad \Longrightarrow \quad T = \frac{2}{3}\mu W \times \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right] = \mu WR$$

where R = Mean radius of friction surface, $R = \frac{2}{3}\left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$

Considering uniform wear

In **Figure (2)** , let ***p*** be the normal intensity of pressure at a distance ***r*** from the axis of the clutch. Since the intensity of pressure varies inversely with the distance, therefore;

p.r. = C (a constant) **or** **p = C/r** **..... (1)**

and the normal force on the ring, $\delta W = P.2\pi r.dr = \frac{C}{r} \times 2\pi r.dr = 2\pi C.dr$

∴ Total force acting on the friction surface,

$$W = \int_{r_2}^{r_1} 2\pi C.dr = 2\pi C[r]_{r_2}^{r_1} = 2\pi C(r_1 - r_2) \quad \Longrightarrow \quad C = \frac{W}{2\pi(r_1 - r_2)}$$

We know that the frictional torque acting on the ring, $T_r = 2\pi\mu.P.r^2.dr = 2\pi\mu.\frac{C}{r} \times r^2 dr$

∴ Total frictional torque on the friction surface,

$$T = \int_{r_2}^{r_1} 2\pi\mu C.r.dr = 2\pi\mu C\left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\mu C\left[\frac{r_1^2 - r_2^2}{2}\right]$$

$$T = \pi\mu.C(r_1^2 - r_2^2) = \pi\mu \times \frac{W}{2\pi(r_1 - r_2)} \times (r_1^2 - r_2^2)$$

$$T = \frac{1}{2}\mu.W(r_1 + r_2) = \mu WR$$

R = Mean radius of the friction surface, $R = \frac{1}{2}(r_1 + r_2)$

Multiple Disc Clutch

A multiple disc clutch, as shown in **Figure (3)** , may be used when a large torque is to be transmitted. The inside discs (usually of steel) are fastened to the driven shaft to permit axial motion (except for the last disc). The outside discs (usually of bronze) are held by bolts and are fastened to the housing which is keyed to the driving shaft. The multiple disc clutches are extensively used in motor cars, machine tools etc.

Let n_1 = Number of discs on the driving shaft, and n_2 = Number of discs on the driven shaft.

∴ Number of pairs of contact surfaces, $n = n_1 + n_2 - 1$

and total frictional torque acting on the friction surfaces or on the clutch,

$T = n.\mu.W.R$

where R = Mean radius of the friction surfaces,

$$R = \frac{2}{3}\left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$$ (For uniform pressure)

$$R = \frac{1}{2}(r_1 + r_2)$$ (For uniform wear)

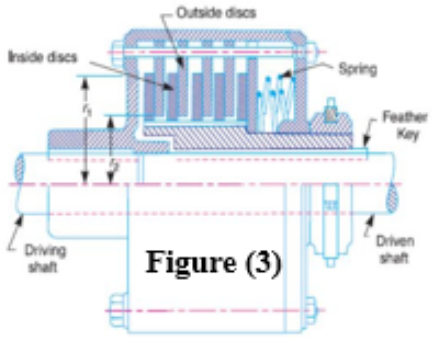


Figure (3)

Cone Clutch

A cone clutch, as shown in **Figure (4)** , was extensively used in automobiles but now-a-days it has been replaced completely by the disc clutch.

It consists of one pair of friction surface only. In a cone clutch, the driver is keyed to the driving shaft by a sunk key and has an inside conical surface or face which exactly fits into the outside conical surface of the driven. Due to the frictional resistance set up at this contact surface, the torque is transmitted from one shaft to another In some cases, a spring is placed around the driven shaft in contact with the hub of the driven.

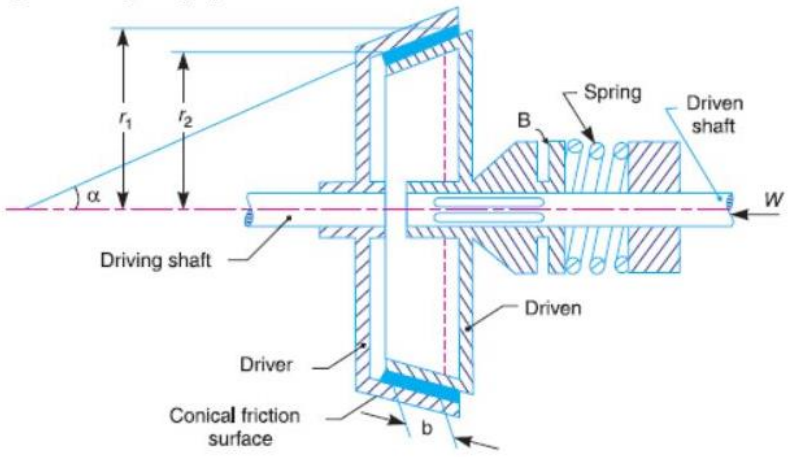


Figure (4)

Consider a pair of friction surface as shown in **Figure (*5*)**. Since the area of contact of a pair of friction surface is a frustrum of a cone, therefore the torque transmitted by the cone clutch may be determined in the similar manner as discussed for conical pivot bearings.

p_n = Intensity of pressure with which the conical friction surfaces are held together (i.e. normal pressure between contact surfaces),

r_1 and r_2 = Outer and inner radius of friction surfaces respectively.

R = Mean radius of the friction surface, $R = \frac{1}{2}(r_1 + r_2)$

α = Semi angle of the cone (also called face angle of the cone) or the angle of the friction surface with the axis of the clutch,

μ = Coefficient of friction between contact surfaces, and

b = Width of the contact surfaces (also known as face width or clutch face).

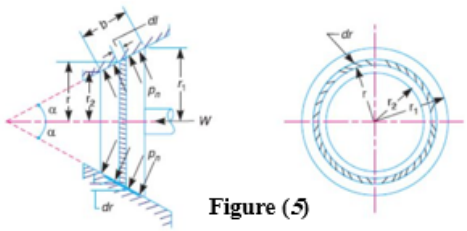


Figure (*5*)

Consider a small ring of radius ***r*** and thickness ***dr***, as shown in **Figure (5)**. Let ***dl*** is length of ring of the friction surface, such that **dl = dr.cosec α**

∴ Area of the ring, **$A = 2\pi$ r.dl = 2πr.dr cosec α**

We shall consider the following two cases :

1. When there is a uniform pressure, and
2. When there is a uniform wear.

Considering uniform pressure

We know that normal load acting on the ring, δW_n = **Normal pressure × Area of ring** = $p_n \times 2\pi$ r.dr.cosec α

and the axial load acting on the ring,

δW = Horizontal component of δW_n (i.e. in the direction of W)

$= \delta W_n \times \sin\alpha = p_n \times 2\pi$ r.dr. cosec α × sin α = $2\pi \times p_n$.r.dr

∴ Total axial load transmitted to the clutch or the axial spring force required,

$$W = \int_{r_2}^{r_1} 2\pi P_n . r\,dr = 2\pi P_n \left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi P_n \left(\frac{r_1^2 - r_2^2}{2}\right) \quad \Longrightarrow \quad W = \pi P_n (r_1^2 - r_2^2)$$

$$p_n = \frac{W}{\pi(r_1^2 - r_2^2)} \qquad .....(1)$$

We know that frictional force on the ring acting tangentially at radius ***r***,

$F_r = \mu.\delta W_n = \mu.p_n \times 2\pi$ r.dr.cosec α

∴ Frictional torque acting on the ring, $T_r = F_r \times r = \mu.p_n \times 2\pi$ r.dr. cosec α.r = $2\pi\,\mu.p_n$ .cosec α.r2 dr

Integrating this expression within the limits from r_2 to r_1 for the total frictional torque on the clutch.

∴ Total frictional torque,

$$T = \int_{r_2}^{r_1} 2\pi\mu.P_n . \operatorname{cosec}\alpha\, r^2 . dr = 2\pi\mu.P_n \operatorname{cosec}\alpha \left[\frac{r^3}{3}\right]_{r_2}^{r_1} = 2\pi\mu.P_n \operatorname{cosec}\alpha \left[\frac{r_1^3 - r_2^3}{3}\right]$$

Substituting the value of p_n from equation (1), we get $T = 2\pi\mu \times \frac{W}{\pi(r_1^2 - r_2^2)} \operatorname{cosec}\alpha \times \left[\frac{r_1^3 - r_2^3}{3}\right]$

$$T = \frac{2}{3}\mu W \operatorname{cosec}\alpha \times \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$$

Considering uniform wear

In **Figure (5)**, let p_r be the normal intensity of pressure at a distance **r** from the axis of the clutch. We know that, in case of uniform wear, the intensity of pressure varies inversely with the distance.

p.r. = C (a constant) **or** **p = C/r**

We know that the normal load acting on the ring,

δW_n = **Normal pressure × Area of ring** = $p_r \times 2\pi$r.dr cosec α

Acti

and the axial load acting on the ring ,

$\delta W = \delta W_n \times \sin\alpha = p_r .2\pi r.dr.\text{cosec}\,\alpha .\sin\alpha = p_r \times 2\pi r.dr$

$$\delta W = \frac{C}{r}\times 2\pi r.dr = 2\pi C.dr$$

∴ Total axial load transmitted to the clutch, $W = \int_{r_2}^{r_1} 2\pi C.dr = 2\pi C\left[r\right]_{r_2}^{r_1} = 2\pi C(r_1 - r_2)$

$$C = \frac{W}{2\pi(r_1 - r_2)}$$

We know that frictional force acting on the ring, $F_r = \mu.\delta W_n = \mu.p_r \times 2\pi r \times dr\,\text{cosec}\,\alpha$

and frictional torque acting on the ring, $T_r = F_r \times r = \mu.p_r \times 2\pi r.dr.\text{cosec}\,\alpha \times r$

$$T_r = 2\pi\mu\frac{C}{r}.r^2.dr\,\text{cosec}\,\alpha = 2\pi\mu C \times rdr\,\text{cosec}\,\alpha$$

∴ Total frictional torque acting on the clutch,

$$T = \int_{r_2}^{r_1} 2\pi\mu C.r.dr\,\text{cosec}\,\alpha = 2\pi\mu.P\,\text{cosec}\,\alpha\left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\mu.P\,\text{cosec}\,\alpha\left[\frac{r_1^2 - r_2^2}{2}\right]$$

Substituting the value of C from equation (1), we have

$$T = 2\pi\mu\times\frac{W}{2\pi(r_1 - r_2)}\text{cosec}\,\alpha\times\left(\frac{r_1^2 - r_2^2}{2}\right)$$

$$T = \frac{1}{2}\mu.W\,\text{cosec}\,\alpha(r_1 + r_2) = \mu WR\,\text{cosec}\,\alpha$$

Where $R = \frac{1}{2}(r_1 + r_2)$ mean radius of friction surface

The forces on a friction surface, for steady operation of the clutch and after the clutch is engaged, is shown in **Figure (6)**

$$r_2 - r_1 = b\sin\alpha \text{ , and } R = \frac{r_1 + r_2}{2} \text{ or } r_1 + r_2 = 2R$$

$$P_n = \frac{W}{\pi(r_1^2 - r_2^2)} = \frac{W}{\pi(r_1 - r_2)(r_1 + r_2)} = \frac{W}{2\pi R\,b\sin\alpha}$$

$W = p_n \times 2\pi R.b \sin\alpha = W_n \sin\alpha$

W_n = Normal load acting on the friction surface = $p_n \times 2\pi R.b$

$T = \mu\,(p_n \times \pi Rb \sin\alpha)\times R\,\text{cosec}\alpha = \pi\mu\, p_n R^2\, b$

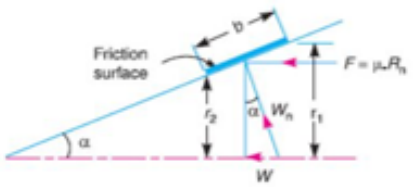
∴ Axial force required for engaging the clutch,

$W_e = W + \mu.W_n \cos\alpha = W_n \sin\alpha + \mu.W_n \cos\alpha$

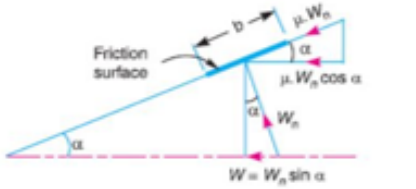
$= W_n(\sin\alpha + \mu\cos\alpha)$

$W_d = W_n(\mu\cos\alpha - \sin\alpha)$

W_d : the axial force required to disengage the clutch



(a) For steady operation of the clutch.



(b) During engagement of the clutch.

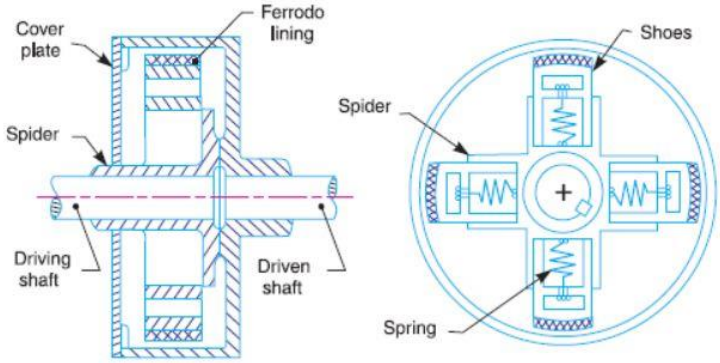
Figure (6)

Centrifugal Clutch

The centrifugal clutches are usually incorporated into the motor pulleys. It consists of a number of shoes on the inside of a rim of the pulley, as shown in **Figure(7)**. The outer surface of the shoes are covered with a friction material. These shoes, which can move radially in guides, are held against the boss (or spider) on the driving shaft by means of springs.

The force with which the shoe presses against the driven member is the difference of the centrifugal force and the spring force. The increase of speed causes the shoe to press harder and enables more torque to be transmitted.

In order to determine the mass and size of the shoes, the following procedure is adopted :



Figure(7)

Mass of the shoes

Consider one shoe of a centrifugal clutch as shown in **Figure (8)**

m = Mass of each shoe,

n = Number of shoes,

r = Distance of centre of gravity of the shoe from the centre of the spider,

R = Inside radius of the pulley rim,

N = Running speed of the pulley in r.p.m.,

ω = Angular running speed of the pulley in **rad/s** = **2πN/60 rad/s,**

ω_1 = Angular speed at which the engagement begins to take place and ,

μ = Coefficient of friction between the shoe and rim.

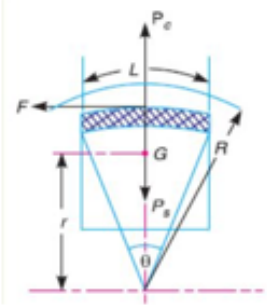


Figure (8)

We know that the centrifugal force acting on each shoe at the running speed, $P_c = m.\omega^2 .r$

and the inward force on each shoe exerted by the spring at the speed at which engagement begins to take place, $P_s = m\,(\omega_1)^2\, r$.

∴ The net outward radial force (i.e. centrifugal force) with which the shoe presses against the rim at the running speed = $P_c - P_s$

and the frictional force acting tangentially on each shoe, $F = \mu (P_c - P_s)$

∴ Frictional torque acting on each shoe, $= F \times R = \mu (P_c - P_s) R$.

and total frictional torque transmitted, $T = \mu (P_c - P_s) R \times n = n.F.R$

From this expression, the mass of the shoes (m) may be evaluated

Size of the shoes

l = Contact length of the shoes,

b = Width of the shoes

R = Contact radius of the shoes. It is same as the inside radius of the rim of the pulley.

θ = Angle subtended by the shoes at the centre of the spider in radians.

p = Intensity of pressure exerted on the shoe. In order to ensure reasonable life, the intensity of pressure may be taken as **0.1 N/mm²** .

We know that $\theta = l/R$ rad or $l = \theta.R$

∴ Area of contact of the shoe, $A = l.b$

and the force with which the shoe presses against the rim, $= A \times p = l.b.p$

Since the force with which the shoe presses against the rim at the running speed is $(P_c - P_s)$, therefore

$l.b.p = P_c - P$ From this expression, the width of shoe (b) may be obtained.